ESSAYS ON THE CONSUMER DEMAND FOR AND OPTIMAL PRICING OF STATE LOTTERY GAMES

A Dissertation

by

MICHAEL ALAN TROUSDALE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2012

Major Subject: Agricultural Economics

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Approved by:

Chair of Committee,	Richard Dunn
Committee Members,	Oral Capps, Jr.
	David Leatham
	Steven Puller
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ABSTRACT

Essays on the Consumer Demand for and Optimal Pricing of State Lottery Games.

(May 2012)

Michael Alan Trousdale, B.A., Brigham Young University

Chair of Advisory Committee: Dr. Richard Dunn

This dissertation is a collection of three economic studies on the demand for and optimal pricing of state lottery games. Lottery betting is a multi-billion dollar industry that provides an important source of government revenue. Since lotteries operate at such a large scale, suboptimal pricing could lead to substantial losses in potential profit. This body of work provides a significant contribution to the literature on lottery demand by introducing a number of innovative modeling techniques that resolve major shortcomings found in current methods and provide direct policy implications for improving the profitability of state lottery games.

The first essay discusses and resolves three important issues widely overlooked in the literature on lottery demand: the treatment of observations with super-unitary expected values, controlling for the endogeneity of price, and the usefulness of estimating price elasticities evaluated at the sample mean. Results indicate that data censoring presents a greater estimation bias than the endogeneity of the effective price. The second essay extends the effective price model of lottery demand into a setting where a single controller operates a portfolio of games simultaneously. Expenditure, own-, and cross-price elasticities for several on-line lottery games are estimated with a Barten synthetic demand system. The elasticity estimates indicate that Texas Lottery games are largely economic substitutes and portfolio profits are not maximized at current prices. Finally, the third essay describes a new method to analyze the profitability of different pricing schemes that explicitly accounts for the intertemporal nature of lottery games with rolling jackpots. Since period-by-period variation in sales induced by rolling jackpots causes changes in the probability that a jackpot is won, which in-turn influences the probability of reaching new drawings with higher jackpot amounts, static analysis of lottery profitability could lead to biased estimates of expected profit. By utilizing a Monte Carlo integration procedure, a measure of expected profit is obtained through the simulation of lottery play over a period of four years. Hypothetical policy changes are examined to estimate potential increases in profitability. Empirical results for the game, Lotto Texas, indicate that a $0.40 increase in price would lead to an estimated increase in profit ranging from $142 million to $191 million over four years.

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1. INTRODUCTION AND LITERATURE REVIEW

1.1 INTRODUCTION

Lottery betting is the most widely available form of legalized gambling in the United States. Available in all but seven states, it provides a significant source of government revenue, totaling over $53 billion nationwide in 2009 (La Fleur 2010). In response to budget shortfalls during the recent recession, interest in lotteries as revenue generators has increased with several states adding new games to their portfolios. Given their scale, even small improvements in the design and pricing of new and existing games could have a substantial impact on government revenue. In addition, lotteries represent the purest form of a risky good. As a result, studying the economics of lottery betting can provide greater insight into the demand for goods with stochastic payouts such as insurance and financial instruments.

Historically, the analysis of demand for lottery games has been conducted primarily within a framework known as the Effective Price Model (EPM), which attempts to model the variation in ticket sales over time in response to changes in the implicit cost of a ticket induced by rolling jackpots. The implicit, or effective, price combines the nominal ticket price (a fixed, certain cost) with a measure of the expected prize return (an uncertain benefit) in order to quantify the value of placing a bet for a

This dissertation follows the style of *American Economic Review*.

particular drawing. This method provides a way to obtain a variable price measure in a market where nominal prices are fixed.

While the EPM has been an important tool for economists to evaluate the profitability of lottery games, a number of methodological limitations still exist. The aim of this dissertation is threefold, with each section building upon the previous. In the first essay, I address a number of important issues that have been overlooked within the current body of literature regarding the empirical estimation of a single game's demand relationship under the effective price model. Specifically, these issues include the treatment of negatively-defined effective prices, the endogeneity of the effective price, and the validity of summarizing price sensitivity using a single weighted-average price elasticity. Improper treatment of these issues could potentially lead to substantially biased estimates in the price elasticity of demand of a lottery game, as well as improper implications on policies aimed at improving a target game's overall profitability.

The second essay extends this analysis to a setting where a single lottery operator manages a portfolio of concurrent games. In the market for lottery gambling, states have moved toward operating multiple games, either by adding new games or joining multi-state coalitions. While many studies have analyzed the demand for single games in isolation, relatively few have modeled the relationship between closely competing games within a portfolio. To accomplish this task, I incorporate the use of a formal demand system, which provides three major advantages over previous approaches: (1) it is consistent with the theory of utility maximization; (2) it allows estimation of both compensated and uncompensated own- and cross-price elasticities; (3) it provides parameter identification for games that exhibit no variation in the effective price. Ultimately, this model integrates the study of goods with stochastic utility (lottery games) with methods more commonly used to examine the demand for traditional consumer products.

In the final essay, I develop a new method to analyze the profitability of different pricing schemes that explicitly accounts for the intertemporal nature of lottery games with rolling jackpots. Previous work treats the profit maximization problem of the lottery controller in a static framework so that changes in price only influence profitability through the effect on period-by-period sales and expected payouts. This situation neglects, however, that changes in the probability that a jackpot is won influences the probability of reaching new drawings with higher jackpot amounts. Thus, while there is little cost in viewing lotteries as repeated static games from the perspective of players, the problem of profit maximization is clearly connected intertemporally because of rollovers. Intuitively, the more the jackpot is rolled over, the higher the ticket sales, but the more the resulting payouts will be when the jackpot is eventually claimed. Because this issue cannot be directly addressed in a static framework, I utilize a Monte Carlo integration procedure to obtain a measure of expected profit through the simulation of lottery play over a period of four years. Such a procedure also provides a way to examine the effects of hypothetical changes in a game's pricing on total profits earned by the lottery controller.

1.2 BACKGROUND ON LOTTERIES

Modern lottery games can be classified into two major categories: off-line and on-line, both of which are typically offered within a state's game portfolio. Off-line games, such as scratch-offs or pull-tabs, are games that do not require the use of a computer terminal for purchase. The tickets for a particular off-line game are printed in a fixed quantity with a duration lasting only as long as there are still tickets to be sold. These games reveal instantly to the player whether they have won a prize and are relatively simple in design, offering a predetermined number of winning tickets. On-line games require the use of a computer terminal to access a network, through which the player's bets are recorded. Although these games come in various forms (e.g. lotto, bingo, keno, and other numbers games), they generally involve having a player select a small group of numbers from a larger set and are awarded prizes based upon how well their selection matches up with a randomly drawn result.

Forty-three states as well as the District of Columbia, Puerto Rico, and the US Virgin Islands operate lotteries. The operation of lotteries by individual states in the United States dates back only to the early 1960's, but grew substantially over the next three decades, peaking in the late 1980's with two-thirds of the nation's population residing in states actively promoting the sale of lottery tickets (Clotfelter and Cook 1989). Following the economic recession of 2008, interest in lotteries has increased as states seek additional revenue sources. This trend is evident in FIGURE 1.1, which depicts the number of states participating in the two major multi-state lotteries in the United States, Powerball and Mega Millions.

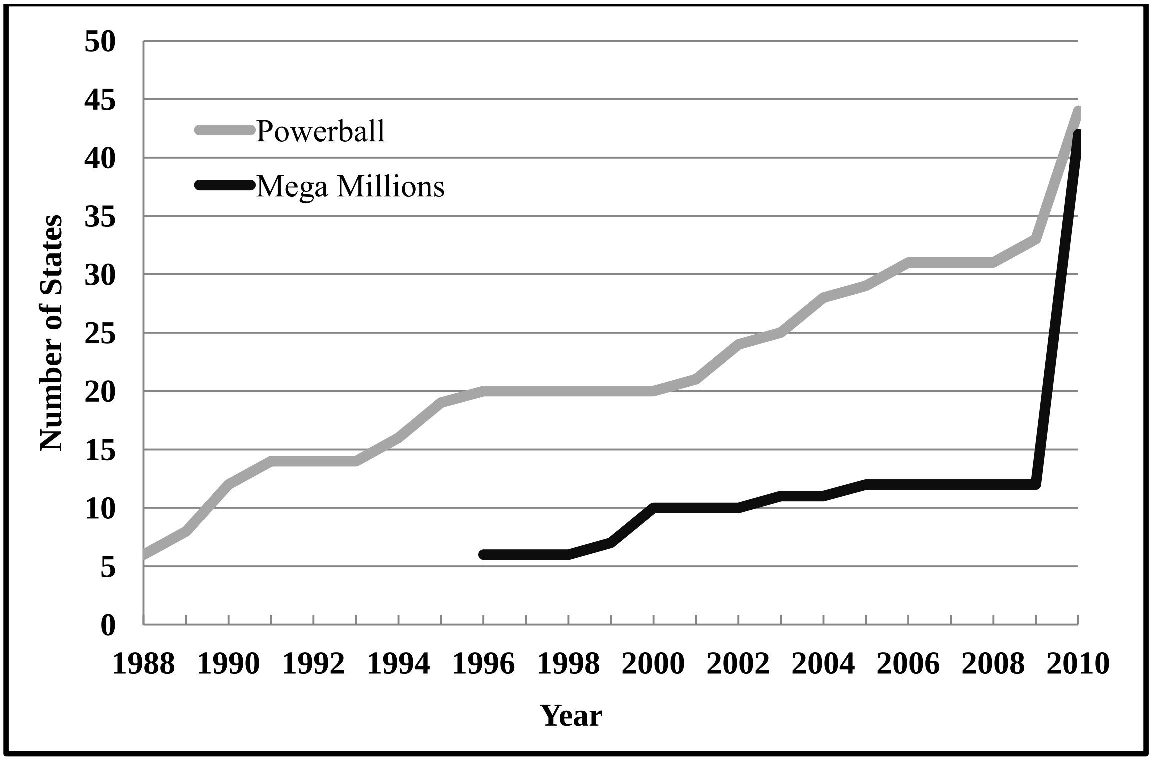

FIGURE 1.1. The Number of States Participating in Multi-State Lotteries by Year

TABLE 1.1. Allocation of State Lottery Proceeds

Public Service	No. of States
Public & Higher Education	36
Health & Human Services	12
General Government	11
Environmental Control	10
Public Safety & Corrections	8
Economic Development	4
Property Tax Relief	2

Although lottery games serve as a form of entertainment, they are established first and foremost as a source of revenue for the jurisdictions that operate them. Depending on the state, revenues collected from the sales of lottery tickets are used to fund a variety of public programs. TABLE 1.1 summarizes general classifications of such programs along with the number of states that allocate lottery funds toward each. According to the Texas Comptroller of Public Accounts, annual lottery revenues amount to approximately 2% of total taxes in Texas (over $1.5 billion), which is more than the revenue collected from individual taxes on cigarettes, alcohol, and several other major goods and services.

Although the Texas Lottery purveys both on-line and off-line games, this paper focuses solely on the analysis of the state's portfolio of on-line lottery games. The period-by-period rolling jackpots and pari-mutuel prize payouts of on-line games provide the price variation necessary for econometric identification. Off-line games do not exhibit such characteristics and cannot be modeled within the framework discussed in this body of work.

1.3 LITERATURE REVIEW

Much of the economic literature of demand for lottery gambling was developed following the explosive growth in this form of public financing throughout the United States in the late 1980's. Cook and Clotfelter (1993) were among the first to estimate the demand for lottery in this period, pioneering the Effective Price Model (EPM). Their method attempts to explain how purchases of lottery tickets vary in response to changes in the expected value of a bet, where the expected value is a function of both a period's ticket sales and the probability of winning. Although an effective price is not explicitly defined, the authors develop a time-series model of demand for lotto in Massachusetts, specifically targeting the association between sales and the game's jackpot, while also controlling for rollovers and the expected value. They compute the elasticity of sales with respect to the jackpot to examine whether there are advantages to increasing the scale of a lotto game, possibly through multi-state consortia.

Building upon this idea, Gulley and Scott (1993) develop a similar model but argue that it is appropriate to define an effective price of a lottery bet. The authors define the effective price of a ticket to be the nominal price ($1) minus the expected value, which exhibits an inverse relationship with the number of tickets sold. Since the true expected value of a pari-mutuel bet is a function of total sales, which can only be determined once all bets have been taken, a proxy variable must be used to approximate players' *ex ante* expectations of total sales. They estimate their model using a two-stage method where the effective price is first regressed on a time trend as well as the jackpots and rollover amounts of competing games in order to obtain an *ex ante* effective price. In the second-stage, log sales are regressed on the *ex ante* log of effective price, with the price coefficient interpreted as the price elasticity of demand evaluated at the sample mean. For the four lotto-style games analyzed, the author's model provides elasticities of -1.15, -1.92, -1.20, and -0.19, respectively, leading them to conclude that given the theoretical revenue maximizing average elasticity of -1.19 (which takes administrative costs and sales commissions into consideration), two of the games are not priced optimally.

A number of papers have built upon this model by adding different controls or applying it to analyze different lotteries. For example, Farrell, Morgenroth, and Walker (1999), and Farrell and Walker (1999) both analyze the UK National Lottery (UKNL) using a variant of the effective-price model (controlling for myopic addiction) to estimate price elasticities, with the latter continuing the line of research by using these elasticity estimates to evaluate the welfare effects of introducing a lottery. The price elasticity they compute for the UKNL is -1.51. Forrest, Gulley, and Simmons (2000) also examine the UKNL lottery using an effective-price model very similar to Gulley and Scott (1993), with an effective price defined as the expected loss (or one minus the expected value) and a log-log regression specification of sales on price. They estimate both short- and long-run price elasticities for the UKNL's first three years of operation and report these values to be -0.66 and -1.03, respectively.

Farrell, Hartley, Lanot, and Walker (2000) analyze UKNL data to determine whether players choose their numbers uniformly and incorporate this information into a model to measure the game's price elasticity. They find that their data exhibit more rollovers than expected given the structure of the game's rules, providing evidence that individuals do not choose their numbers uniformly. They show that numbers above 31 tend to be less popular, presumably since it is common for players to rely upon important dates such as birthdays or anniversaries to select their numbers.

While these papers focus primarily on the analysis of a single game, relatively few studies have tried to examine demand for more than one game simultaneously. Since many states offer multiple concurrent games, it is reasonable to expect the relative effective prices of competing games to influence the choices of players. Purfield and Waldron (1999) examine the substitutability/complementarity between lotto and fixed-odds betting in Ireland. Using an effective price model, their results suggest a complementary relationship between the two types of lottery games. Forrest, Gulley, and Simmons (2004) evaluate the extent to which games in the UKNL are substitutes or complements using the EPM. For each game in their sample, they regress sales on the own effective price as well as the effective prices of other games. They conclude that UK lottery games operate independently of each other, with little evidence to suggest any cross-price effects.

Analyzing data from three US states, Grote and Matheson (2006) compare the sales relationship between a state-run lotto game and Powerball, a multi-state lotto game. They develop a simple log-form model, regressing sales on the jackpot and a dummy representing the addition of the multi-state game. They conclude that while the addition of a new multi-state game tends to cannibalize the sales of smaller single-state games initially, the two appear to exhibit a complementary relationship thereafter. Forrest, Gulley, and Simmons (2010) analyzes the relationship between lottery play and bookmaker betting, which includes horse racing, soccer, and numbers betting. They argue that these two categories of gaming are quite different from one another for a number of reasons. First, while many betting games require some skill for players to seek out value, lotteries, on the other hand, are a game of pure chance. Second, betting games tend to offer much higher expected returns with lower skewness than lotteries. Third, the size of their markets differs greatly, with betting markets only attracting a small fraction of the population. The authors find that according to the data gathered from bookmakers in the UK as well as the UK National Lottery, there is evidence that players tend to substitute away from bookmaker betting when the expected return to the lottery is unusually high (due to rollovers or special draws).

A more recent paper by Perez and Forrest (2011) takes the approach of measuring own- and cross-price elasticities to determine the potential relationship competing games have with each other, using data collected from a portfolio of three lotto games in Spain. The authors use a log-log effective price model estimated equation-by-equation for each game in a fashion similar to Forrest, Gulley and Simmons (2004). They find that own-price elasticities are uniformly less than negative one and argue that since the take-out rates in Spain are more generous to the consumer than in other jurisdictions, this result is not surprising. The authors also find that the cross-price estimates are small and not statistically significant. They attribute this finding to the ability of the lottery operator to minimize the extent to which the sales are cannibalized. This evidence substantiates the claim that lottery games can be treated as unrelated goods.

2. ADDRESSING SOURCES OF ESTIMATION BIAS IN THE EFFECTIVE PRICE MODEL OF LOTTERY DEMAND

2.1 INTRODUCTION

Lottery betting exhibits a number of important characteristics that differentiate it from other, more traditional, economic goods. These characteristics have required researchers to develop unique strategies for modeling demand behavior. When an individual purchases a lottery tickets, they pay a fixed nominal price in exchange for a small chance at winning a large sum of money. Thus, the utility of the transaction is *ex ante* uncertain. Furthermore, the price of a ticket is fixed and cannot be used to predict the time-variation in sales according to a traditional price/quantity demand relationship. The effective price model (EPM) pioneered by Cook and Clotfelter (1993) and Gulley and Scott (1993) provides an innovative approach to model demand behavior in the presence of these issues.

The EPM addresses the problem of uncertainty by measuring the expected value of a lottery bet as a function of total ticket sales as well as the odds and prize amounts for each available prize tier. This measurement provides the expected return a player receives in exchange for paying the nominal ticket price. In order to resolve the issue of identifying a demand relationship under a fixed price, the drawing-by-drawing variation in ticket sales is measured in response to rollover-induced changes in the expected value. Since a ticket purchased in a relatively high-jackpot drawing has a greater value than one purchased in a relatively low-jackpot drawing (due to its higher expected value), an effective price can be defined to reflect the difference in value between drawings with different expected values. According to the EPM of Gulley and Scott (1993), the effective price for a given drawing is defined to be the difference between the fixed nominal price and the variable expected value. Thus as the jackpot increases, the effective price of a ticket decreases because the difference between the nominal price and the expected value gets smaller. Since ticket sales tend to be greater in drawings with higher jackpots, the demand curve can be modeled as the relationship between the quantity of tickets sold and the effective price.

In order to estimate the demand relationship econometrically, the EPM provides a rather straightforward approach. Log total ticket sales are linearly regressed on the log effective price, allowing the estimated coefficient to be interpreted as the price elasticity of demand. This elasticity then can be used to evaluate whether the lottery game is optimally priced. Since the effective price defined above also represents the average profit per ticket, an effective price elasticity of -1 would correspond to a situation where a 1% increase in the effective price would be equally offset 1% decrease in sales, or the point of maximum average profit. According to the model, comparing the proximity of the game's estimated price elasticity to -1 will determine whether the game is optimally priced.

Despite its popularity over the last 20 years as the standard model for analyzing demand for lottery tickets, the EPM still exhibits potentially important shortcomings. The aim of this essay is to address three methodological issues that if not properly addressed could cause biased estimates of the effective price elasticity: (1) the treatment of observations exhibiting super-unitary expected values, (2) controlling for the endogeneity of the effective price, and (3) whether reporting a single price elasticity estimated at the sample mean accurately summarizes behavior along the entire curve.

In order to address the treatment of observations with super-unitary expected values, I approach the problem from a new angle by redefining the effective price concept in a way that has better properties for estimating demand elasticities. Again, previous work has typically defined the effective price as the difference between the nominal price, P and the expected value, EV. Since P minus EV equals the expected profit per ticket, this approaches the demand relationship from the perspective of a profit-maximizing lottery controller.

Despite this definition's intuitive appeal, drawings with large jackpots can exhibit expected values that exceed the nominal cost of a ticket. In such cases, the effective price is negative and thus undefined under the logarithmic econometric specifications that are commonly used in this line of research. These observations are particularly important because they arise when the lottery controller not only sells the most tickets but also must pay out the most money if the jackpot is won. Thus, failing to include them in estimation of the model could have a large impact on estimation results. One proposed solution is to include these observations after censoring the effective price above zero (Perez and Forrest 2011), but for games where super-unitary expected values are not uncommon, the bias from censoring could remain problematic. To address this issue, I explore two alternative modeling approaches: (1) the use of a semi-log specification and (2) the use of a new effective price definition that approaches the problem from the perspective of consumers who are purchasing chances to win *P* dollars. Every ticket purchased is *EV* chances at an effective price of *P/EV* per chance. This alternative price definition is positive under the entire range of expected values, hence, it is possible to compare estimation results from samples that omit or include super-unitary expected values.

Second, I develop a new method to address the potential endogeneity of the effective price. Endogeneity arises because the winner's expected share of the prize pool depends upon the total number of sales. The effective price depends upon the player's prediction of these sales, but only actual sales are observed in the data. Previous studies treat this problem by estimating a first-stage regression of the effective price on exogenous market indicators known to players before the drawing takes place, such as sales under similarly-sized jackpots, the rollover amount of the game, and the jackpots of competing games. The predicted values of the effective price from the first-stage regression then are inserted into the second stage, where sales are regressed on the effective price. To circumvent the need for a two-stage method, I explore the use of a publicly available source of sales prediction information created by the lottery controller itself and published on its website. Since players have easy access to these predictions, which are highly accurate, I argue that they provide a less noisy approximation to the priors formed by individual players.

Third, I use non-parametric methods to estimate the demand relationship between effective price and quantity. Previous research using a log-log regression framework estimates a single weighted-average price elasticity. This may serve as an appropriate statistic for goods that do not exhibit much price variation; however, lotto games experience wide fluctuations in the effective price. Reporting a single elasticity to summarize price sensitivity may obscure important attributes of the demand curve; therefore I employ a non-parametric locally weighted scatterplot smoothing (LOWESS) estimator to determine whether the data exhibit different elasticities along the demand curve and discuss the implications of this result in terms of optimal pricing.

To preview the main empirical results using data on sales for the Texas-operated game Lotto Texas from 2006 to 2009, I find that omitting drawings with super-unitary expected values introduces bias that is six-times greater than that from ignoring the potential endogeneity of effective price. This bias arises because high-jackpot games tend to yield large negative expected profit. In addition, the demand elasticity varies significantly across different modeling specifications; nevertheless, the estimated elasticity of sales with respect to expected profit per ticket is everywhere inelastic, which would suggest that profits would increase by increasing the price of a lotto ticket.

2.2 METHODOLOGY

2.2.1 Data Censoring Problem

In the market for on-line lottery games, players observe two different prices. First is the nominal (explicit) price of a lottery ticket, usually $1, and is what the player actually exchanges in return for a chance at a cash prize. Second is the effective (implicit) price, which is derived from the expected value of a bet, given the odds of winning, the payouts to the winners, and the likelihood of a tie where the prize is split. The implicit value of purchasing a ticket changes from drawing to drawing as successive rollovers increase the value of a bet or as resetting the jackpot (in the event of a win) decreases the value of a bet. Modeling demand in terms of the effective price describes the effect that this implicit cost has on the number of tickets players choose to purchase. This approach is better suited econometrically since the use of rolling jackpots provides substantial variation in the effective price over time, creating an identifiable market demand curve. The equilibrium quantity for lottery tickets is completely determined by demand and does not in any way confound estimates of demand-side relationships.

Although the effective price serves as an artificial means by which to explain the variation in sales, it should still permit reasonable economic interpretations rather than be an *ad hoc* specification that happens to have an inverse relationship with sales. The most common definition used in the literature (see Gulley and Scott (1993); Scoggins (1995); and Forrest, Gulley, and Simmons (2000)) is

$$A = P - EV. \quad [2.1]$$

This approach is more aptly suited to modeling demand from the perspective of the lottery controller. By directly factoring in the payouts to the players, expression [2.1] mathematically represents the average profit per ticket to the lottery controller and can be used to measure total profits by multiplying it by the number of tickets sold. However, when this definition is used in a log-log econometric framework (which is the most common specification used in the literature), the model may suffer from a substantial methodological problem. When the jackpot of a lotto game gets sufficiently large, it is possible for the expected value of a bet to actually exceed the nominal price of a ticket. When this situation happens, the effective price is negative and thus the log transformation is not defined.

One approach to solve this problem would be to simply omit these observations from the sample, but it is clear that this omission will result in biased estimates. Perez and Forrest (2011) propose that the expected values of these observations could be censored just below the nominal price. Thus for a nominal price of $1, all observations that meet or exceed $1 should be censored at $0.99. Again, if the number of data points that need to be censored is relatively large, then the prospect of biased estimates is still a valid concern. Since there is no way to test the degree of bias within the current framework, an entirely new framework is needed to explore this problem further.

In response, I propose two alternative modeling strategies that can be used to address this problem. First, demand can be estimated using a semi-log framework, where log sales are regressed on an absolute measure of the effective price. In this case, the slope coefficient measures the relative change in sales for a given absolute change in the value of the effective price. Second, I explore the option of redefining the effective price in a way that does not yield negative values. This will allow the log-log framework to be maintained.

The most straightforward approach to address the data censoring issue would be to adapt a semi-log regression framework. Under a semi-log specification, the regression model becomes:

$$\log Q = \beta_0 + \beta_1 trend + \beta_2 A + \beta_3 A_{other} + \mu, \qquad [2.2]$$

where Q is the total tickets sales, *trend* is a time trend, A is the effective price defined in [2.1], A_{other} is the effective price of a bet for a competing lotto game, and μ is the error term. The coefficient of interest, β_2, measures the percentage change in sales in response to an absolute change in the average profit per ticket A. While the price coefficient does not represent an elasticity, the price elasticity evaluated at the sample mean can be obtained by multiplying β_2 by the sample mean of A.

Should it be desirable to maintain the log-log framework, the data censoring problem can be resolved by redefining the effective price in a way that does not results in negative prices for observations with super-unitary expected values. Consider the following definition:

$$p = \frac{P}{EV}. \qquad [2.3]$$

Given a typical nominal price of \$1, this definition measures the price as the cost of a chance to win a dollar. To illustrate, consider a drawing where the expected value is computed to be \$.50. In this case, the player would expect to pay two dollars on average for every dollar they win. Thus the effective price to the consumer for such a bet would be \$2. Not only is *P/EV* inversely related to quantity,[1] it does not impart negative values for observations with super-unitary expected values. Defining the effective price in this manner does not directly incorporate the lottery controller's costs associated with prize payouts, this method is thereby more aptly described as modeling price from the perspective of the consumer. Therefore any resulting analysis using this definition will appeal directly in terms total revenue, and not total profits. While expression [2.3] represents the true implicit cost to the consumer, it does not change the fact that players are still only paying \$1 for each bet they make in a given drawing. Total revenue is obtained by multiplying total ticket sales by the nominal price per ticket, or

$$TR \equiv P \times Q. \qquad [2.4]$$

Expression [2.3] is an identity that must be preserved in the face of converting from the nominal price to the effective price. Therefore, in order to preserve this identity, the nominal quantity Q must be redefined as well. Multiplying Q by the expected value or

$$q = Q \times EV \qquad [2.5]$$

provides the necessary "effective" quantity, q. Expression [2.6] mathematically illustrates how p and q preserve the total revenue identity:

$$p \times q = \left(\frac{\$P}{EV}\right) \times \left(Q \times EV\right) = P \times Q \equiv TR\ . \qquad [2.6]$$

The effective quantity is not without an economic interpretation, representing EV purchased chances to win a dollar.

[1]Garrett and Sobel (2004) use 1/EV in their demand model (P = \$1) to preserve the inverse relationship between price and sales with a footnote indicating the use of 1-EV provided similar results. However, it is not made clear why the authors chose the former over the latter.

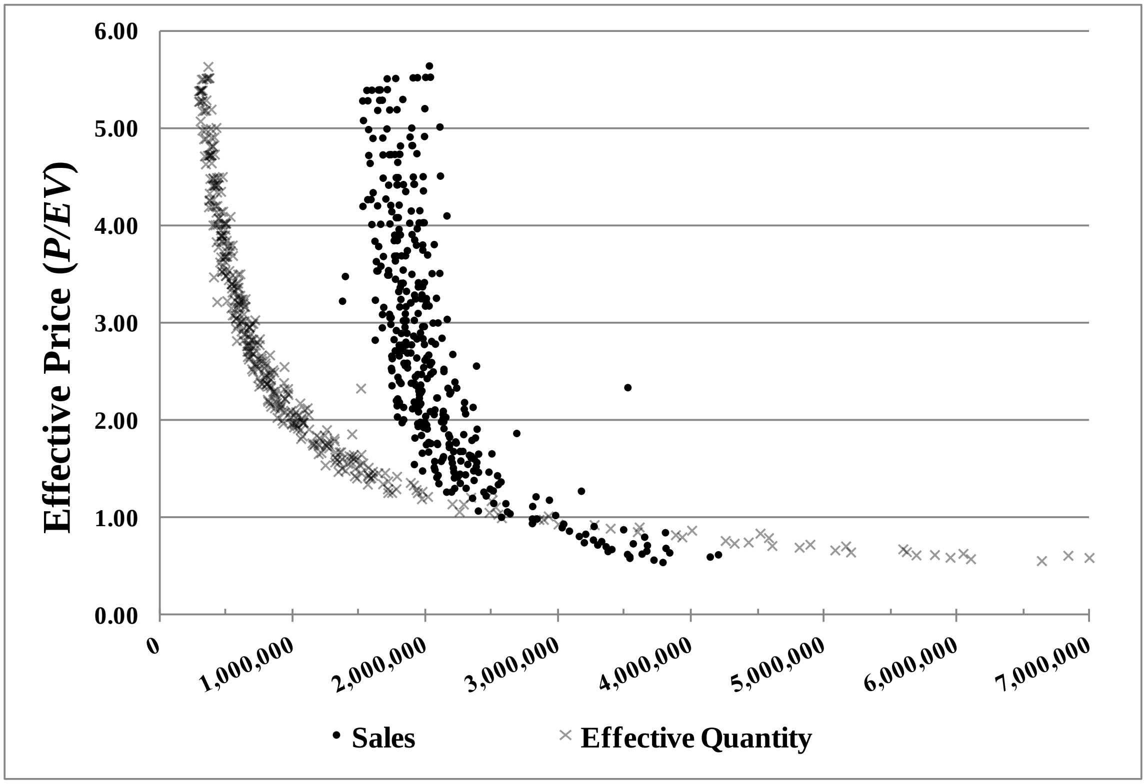

FIGURE 2.1. Scatter Plot of Effective Price against Both Nominal and Effective Quantities for Lotto Texas

FIGURE 2.1 graphically illustrates this transformation using two superimposed scatter plots of price and quantity for 385 independent drawings of the game Lotto Texas. The right-most plot (marked by black dots) graphically depicts the relationship between the effective price (p) against the nominal quantity (Q). The left-most plot (marked by gray x's) depicts the relationship between the effective price (p) against the effective quantity (q). Transforming sales in this fashion provides the economically correct relationship between price and quantity using the definition of P/EV for the effective price. It is important to note the highly non-linear relationship between the effective price and the effective quantity.

As discussed earlier, preceding papers in the literature predominantly rely upon the effective price definition of *P–EV* in a log-log econometric model to estimate the effective price elasticity. The model is given by

$$\log Q = \beta_0 + \beta_1 trend + \beta_2 \log A + \beta_3 \log A_{other} + \mu, \qquad [2.7]$$

where Q is the total tickets sales, *trend* is a time trend, A is the effective price, A_{other} is the effective price of a bet for a competing lotto game, and μ is the error term. The coefficient of interest, β_2, measures the percentage change in sales in response to a one percent change in the average profit per ticket A. Since this specification does not allow a direct test of the degree of bias incurred by censoring the data, expression [2.7] is modified to suit the new effective price definition of $p = P/EV$:

$$\log q = \beta_0 + \beta_1 trend + \beta_2 \log p + \beta_3 \log p_{other} + \mu, \qquad [2.8]$$

where q represents the total effective quantity defined in expression [2.5]; *trend* is a time trend, p is the effective price, p_{other} is the effective price of a bet for a competing lotto game, and μ is the error term. In this case, β_2 provides the mean elasticity of the effective quantity q with respect to the consumer's effective price p. This specification does allow a comparison of results obtained from the full sample against those obtained from omitting or censoring the super-unitary expected value observations.

2.2.2 Expected Value of a Lottery Bet

Typically an on-line lottery game will offer a grand prize (jackpot) along with several smaller prize tiers that are awarded based upon how well a winning player's selected numbers match up to an official, random draw. For large games, the probability of winning the jackpot is extremely small, since it requires the matching of all numbers perfectly (for example matching 6 numbers randomly chosen from a field of 54 yields odds of 1:25,827,165). However, this situation gives the game operator the ability to offer very large base jackpots, which can range from hundreds of thousands of dollars to hundreds of millions of dollars, depending on the size and popularity of the game. If a game employs a rollover, this characteristic allows for even larger jackpots since the amount allocated to the top prize will be moved into the next drawing's pool in the event the top prize is not won. In the case of a tie (i.e. more than one player's numbers exactly match the official draw) pari-mutuel rules require the splitting of the prize pool among all winners. This leads to an interesting trade-off in the face of increased ticket sales; on one hand, as more people play, the larger will be the amount a player stands to win, but on the other hand, it increases the chances a tie will occur (Cook and Clotfelter, 1993). This situation can be true even if jackpots are determined and advertised a period ahead, because the expected value also is a function of lower-tier prizes whose prize pools may be set as a fixed percentage of total sales.

The expected value of a winning the jackpot on a single lottery bet can be expressed by the following equation

$$EV_{jackpot} = (probability)\,x\,(jackpot)\,x\,(share), \qquad [2.9]$$

which represents the probability of matching all numbers multiplied by both the jackpot amount and the winner's expected share of the jackpot. Cook and Clotfelter (1993) point out that under the assumption that the numbers chosen by players are uniformly distributed across the set of all players[2], the expected share can be approximated using a Poisson approximation to the binomial distribution for the probability of observing x winners in N independent random trials with probability π of success. Under the Poisson approximation, the expected value winning the jackpot on a single bet becomes

$$EV_{jackpot} = \theta \cdot jackpot \cdot \left[\sum_{x=0}^{N} \frac{1}{1+x} e^{-\theta N} \frac{(\theta N)^x}{x!} \right], \qquad [2.10]$$

which can be simplified further to

$$EV_{jackpot} = \frac{jackpot}{N} \left(1 - e^{-\theta N}\right). \qquad [2.11]$$

where N is total ticket sales of all other players and θ is the probability of winning the jackpot. Computing the expected value for the lower-tier prizes is done in a similar fashion. The total expected value of a bet is simply the sum of the EVs over all prize tiers, or

$$EV = EV_{jackpot} + EV_{other}. \qquad [2.12]$$

The share of the total expected value of a ticket made up by the lower-tier prizes is much smaller compared to that of the jackpot. Since the state takes a significant share of the ticket sales, this value is usually less than one dollar, but grows as rollovers increase the jackpot. It is even possible for a lottery to occasionally experience expected values

[2] Farrell, Hartley, Lanot, and Walker (2000) argue that this assumption does not necessarily hold. According to their lottery data, they find far more rollovers than expected, which suggests that individuals may not pick numbers at random. This situation can happen when, for example, individuals rely frequently upon choosing numbers based on the dates of special events (e.g. birthdays & anniversaries), perceived "lucky" numbers (e.g. 7 & 11), or shy away from perceived "unlucky" numbers (e.g. 13). Thus, in the face of non-uniform number choice (or conscious selection), the use of the Poisson approximation for the expected share calculation may be slightly biased. I recognize the potential for bias in assuming a uniform distribution of numbers across players in a given drawing, however the data do not allow me to control for conscious selection.

greater than one if enough periods go by without the sale of a winning ticket, causing a bet to become more than fair to the player.

2.2.3 Endogeneity of the Effective Price

It is evident that in the context of lottery games, a regression of quantity on the effective price will suffer from endogeneity. Endogeneity arises because the effective price is a function of total sales, which are not determined until after sales have closed for a drawing. Since the true effective price is not knowable before all bets have been taken, players must form an expectation of the post-drawing total sales in their purchasing decisions. Players' expectations are unobservable and must be approximated. Previous papers (including Gulley and Scott (1993); and Forrest, Gulley, and Simmons (2000)) attempt to remedy this problem by running a two-stage estimation procedure. In the first stage, the *ex post* price (computed using sales *ex post* of the drawing) is regressed on all relevant observable information that was likely available to players at the time of purchase, before the occurrence of the drawing. Such information includes prior rollover amounts, prior sales under similarly sized jackpots, etc. The predicted values of the first-stage regression then are included in a second-stage regression specified according to expression [2.7] above.

While the two-stage method attempts to take advantage of information made available before a given drawing takes place, it still suffers from the problem of omitted variables. The dataset used in this study provides a unique and more accurate remedy. In Texas, on-line games with advertised jackpots are managed according to a specific timeline. Before sales begin for a particular drawing, the Texas Lottery Commission (TLC) sets a fixed value for the jackpot of the upcoming drawing. This value is advertised for the duration that sales are active, usually until just before the drawing actually occurs. Since the advertised jackpot is set beforehand, the TLC must make a prediction as to how many tickets will be sold for the upcoming drawing in order to ensure there will be the necessary funds to support the jackpot. This prediction is made according to their own model that factors in the sales of recent drawings, as well as any other market conditions that are known to affect sales, such as holidays, natural disasters, or even dates that players may likely perceive to be lucky (e.g. 7/7/2007). The exact specification of the prediction model is proprietary and incorporates information that is not likely observable to the econometrician. However, their sales predictions are made publicly available on the TLC's website **before** sales are opened for the corresponding drawing. Assuming either the TLC's prediction model reasonably approximates a representative player's prediction or that a representative player monitors the TLC's prediction and uses it for their own, then this value may used to compute the *ex ante* effective price, entirely avoiding the need for a two-stage model that relies upon a weaker set of information.

2.2.4 Use of a Mean-evaluated Price Elasticity

The use of a single price elasticity evaluated at the sample mean is a common practice in demand analysis. Since a price elasticity is only a local measure of price sensitivity, reporting the elasticity at the sample mean works well for goods that experience only small changes in prices. In the case of lottery betting, the presence of rollovers creates substantial variation in the effective price making it harder to justify the reporting of only a single estimate. To illustrate, I refer back to the Lotto Texas example. Using the effective price definition of Gulley and Scott (1993) according to expression [2.1], according to 385 independent drawings from April 2006 to Dec 2009, Lotto Texas experiences a range of effective prices from a high of $0.83 to a low of -$0.86 (if we allow the price to remain uncensored), with a sample mean of $0.52. The data are also heavily skewed, with the bulk of the observations occurring at the high-price/low-sales side. On the other hand, even though fewer observations occur at the low-price side, the number of sales at these points is much higher. The same argument can still be made when moving to the consumer analogue of a demand curve based on the relationship between the new effective price (expression [2.2]) and the effective quantity (expression [2.4]). If thought in terms of average profit to the lottery controller, it is reasonable to suspect that drawings occurring on one side of the demand curve may be more lucrative than drawings occurring at the other. Therefore basing conclusions about the game's overall performance on a single statistic appears to be misguided.

In the literature on demand for lottery betting, the use of a log-log specification not only compels the estimated price coefficient to be interpreted at the sample mean, but also suffers from an imposed curvature restriction that demand is everywhere iso-elastic at that value, which does not allow for the elasticity to change as you move along the curve. To explore the potential for the locus of points along a lottery demand curve to exhibit different elasticities, I move away from a parametric estimator that relies upon a specific functional form and employ a non-parametric locally weighted scatterplot smoothing (LOWESS) estimator. LOWESS smoothing (Cleveland (1979); Cleveland and Devlin (1988)) is a locally weighted polynomial regression, wherein each point in the sample is fitted to a local subset of the data by a low-degree polynomial using weighted least squares (giving higher weight to the points that are closer to the point whose response is being estimated). LOWESS provides a highly flexible smoothing estimate and is ideal for use on datasets that are densely sampled. This characteristic is valuable because it avoids imposing a rigid structure on the curvature of the lottery data.

The LOWESS smoother was used obtain two different demand curve estimates. FIGURE 2.2 illustrates the use of a LOWESS regression of sales quantity (Q) on the average profit per ticket (effective price [2.1]) for the game Lotto Texas. The gray dots represent a mapping of the effective price against quantity. The black line represents the smoothed LOWESS fitted values. These values were estimated using STATA at a bandwidth setting of 0.8.

Using the LOWESS predicted values as the fitted demand curve, arc elasticities were computed at each individual point to determine the degree of variation along the curve. The arc elasticities are mapped against their respective effective prices in FIGURE 2.3. In this figure, the gray dots denote the computed arc elasticity. The black dots designate a second LOWESS smoothing which was used to help identify the underlying trend over the effective price range. According to the data, it is clear that the arc elasticity at each point does not remain constant along the demand curve, but actually appears to become more elastic as the effective price decreases, with a slight upward turn beyond $0.70.

This analysis was replicated over the alternative effective price definition of $p = P/\mathrm{EV}$ (expression [2.3]) as well. FIGURE 2.4 illustrates the use of a LOWESS regression of the effective quantity q on the new effective price p using a bandwidth of 0.8. Again, the gray dots represent a mapping of the effective price/effective quantity pairs, while the black dots denote the LOWESS fitted values. Similarly, the arc elasticities are computed at each point along the demand curve and plotted against the effective price in FIGURE 2.5. The black dots in the figure denote the LOWESS fitted arc elasticities in order to reveal the underlying trend as the effective price varies. Both FIGURES 2.3 and 2.5 provide substantial evidence that the price elasticity is not constant along the demand curve, making it hard to not only justify the dependence upon a single price elasticity to summarize the demand for this game, but to restrict the model to fit an iso-elastic curve.

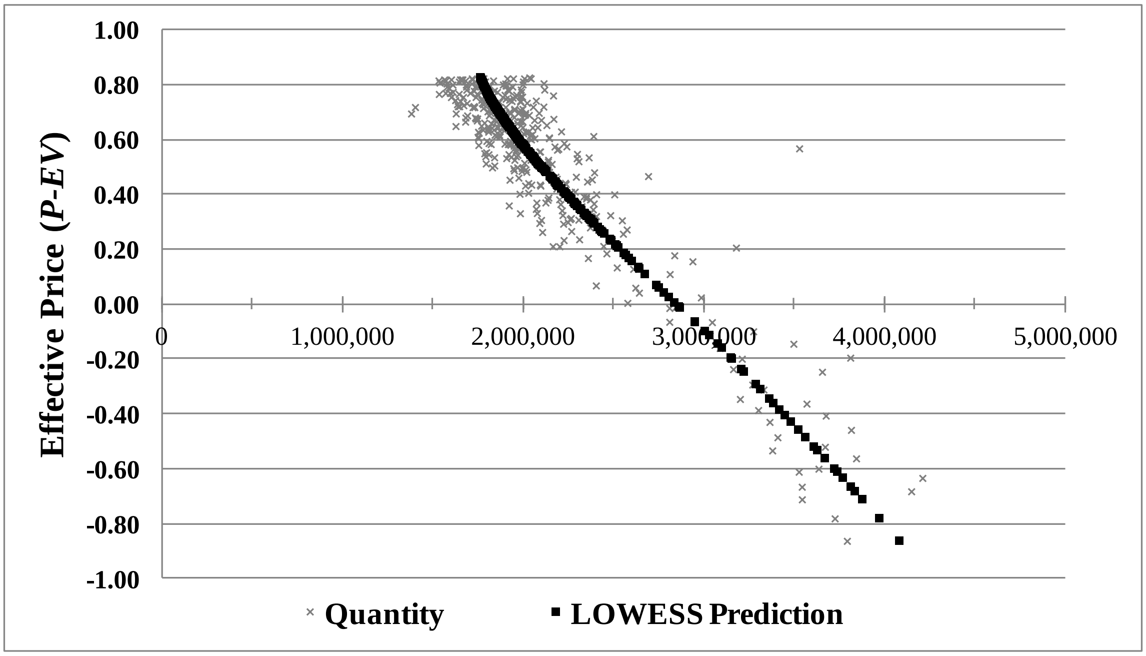

FIGURE 2.2. LOWESS Smoothing of Effective Price ($A = P - EV$) and Quantity (Q) of Lotto Texas

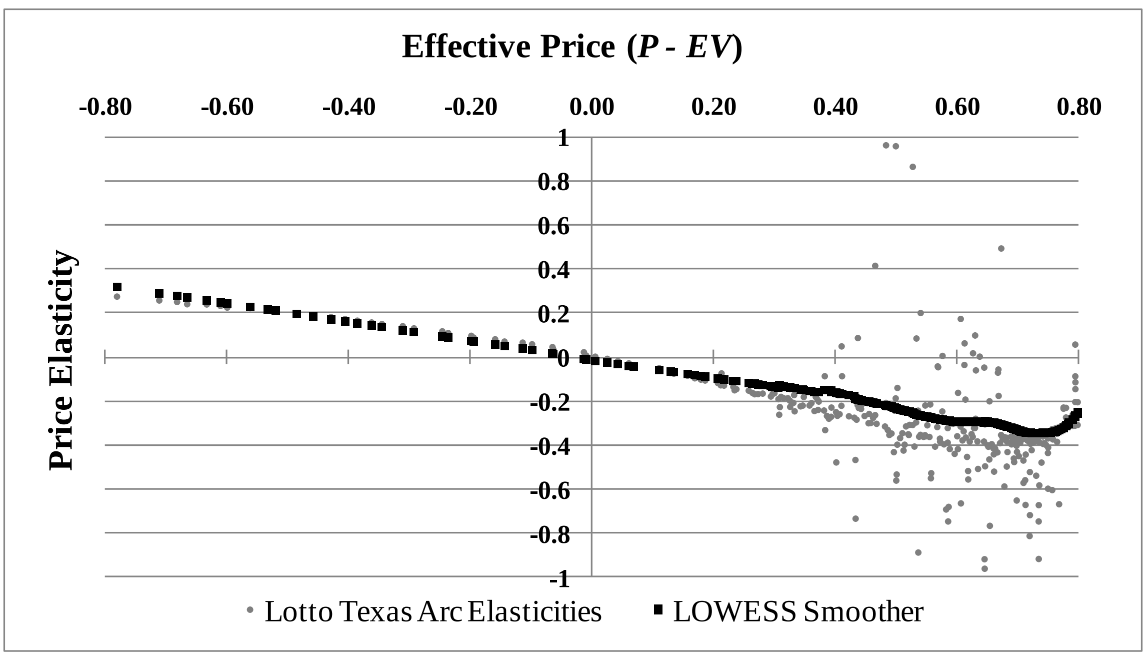

FIGURE 2.3. LOWESS Smoothing of Lotto Texas Arc Elasticities under the Effective Price of $A = P - EV$

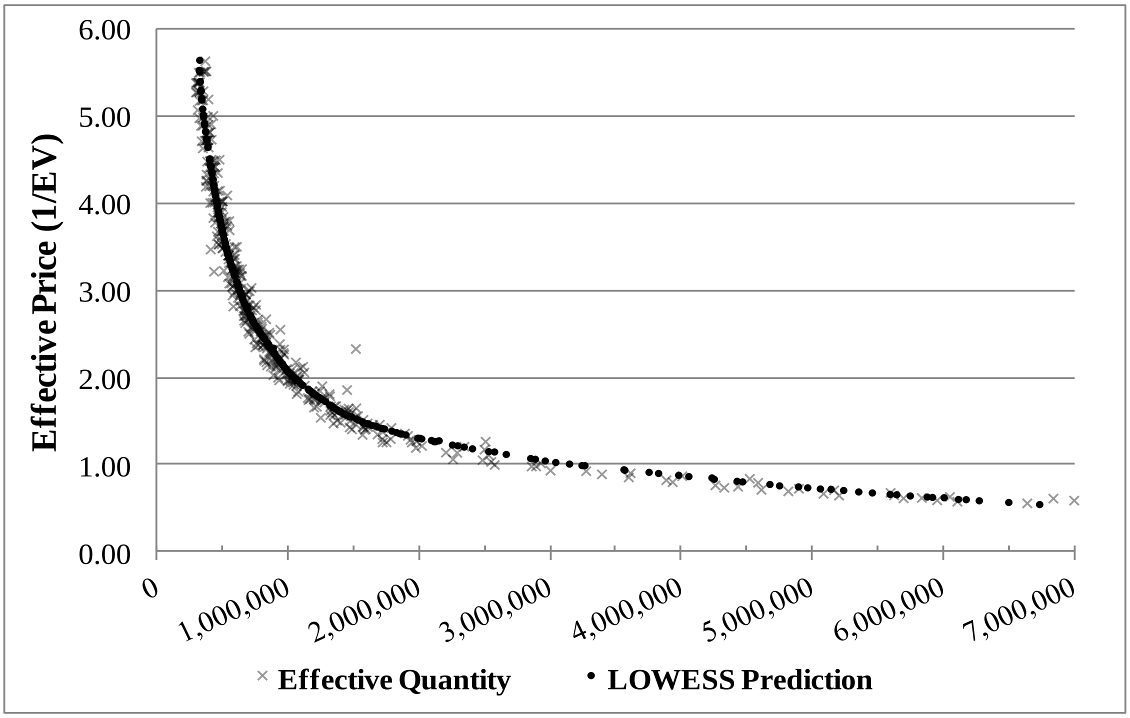

FIGURE 2.4. LOWESS Smoothing of Effective Price ($p = P/EV$) and Effective Quantity ($q = Q/EV$) of Lotto Texas

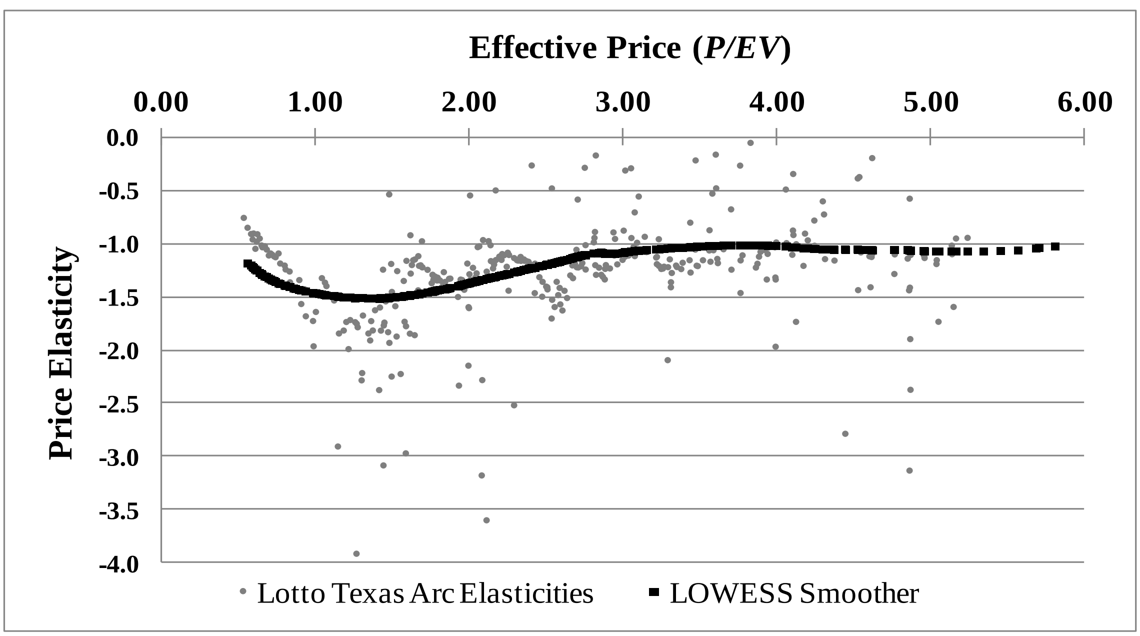

FIGURE 2.5. LOWESS Smoothing of Lotto Texas Arc Elasticities under the Effective Price $p = P/EV$

2.2.5 Profit Maximization Problem

In addressing the issue of improving lottery profitability through price analysis, much of the literature appears to be concerned with obtaining price elasticities in order to determine the optimal direction of price changes. To do this, a number of preceding lottery demand studies, including Gulley and Scott (1993); Forrest, Gulley, and Simmons (2000); and Perez and Forrest (2011) appeal to the results of a static optimization problem of a lottery controller's profit for a single game. Total profit is obtained by subtracting total expected payouts, *X*, from total revenue,

$$\pi = P \cdot Q - X \ . \qquad [2.13]$$

With the nominal price *P* fixed at $1, algebraic manipulation of expression [2.13] yields profit in terms of the per-ticket average profit from expression [2.1]:

$$\begin{aligned} \pi &= Q \cdot \left(1 - \frac{X}{Q}\right) \\ &= Q \cdot (1 - EV) \\ &= Q \cdot A. \end{aligned} \qquad [2.14]$$

For notational simplicity, let 1–*EV* be denoted by *A*. Since *Q* is a function of *A*, the objective function becomes:

$$\max_{A} \pi(A) = Q(A) \cdot A \qquad [2.15]$$

Taking the derivative of profit with respect to the *A* yields:

$$
\begin{aligned}
&Q + A\frac{\partial Q}{\partial A} = 0 \\
&\Rightarrow -\frac{\partial Q}{\partial A} A = Q \\
&\Rightarrow \frac{\partial Q}{\partial A}\frac{A}{Q} = -1 \\
&\Rightarrow \eta_A^* = -1.
\end{aligned} \qquad [2.16]
$$

This result implies that average profit is maximized if the elasticity of sales with respect to A is equal to negative one, at which point a one percent increase in the average profit per ticket is exactly offset by a one percent decrease in the quantity of tickets sold. Preceding papers compare their estimated effective price elasticities to this optimal value as a test for the optimal pricing of a lottery game.

Since this paper explores the consumer analogue of the lottery controller's effective price problem, the objective function needs to be re-specified according to the new effective price/quantity definitions from expressions [2.3] and [2.5]. As I have shown through expression [2.6] above, total revenue is simply the product of p and q. Thus total profit is obtained by subtracting the total expected payout q from total revenue:

$$
\begin{aligned}
\pi &= p \cdot q(p) - q(p) \\
&= \frac{1}{EV} \cdot (sales \cdot EV) - sales \cdot EV \\
&= 1 \cdot sales - sales \cdot EV \\
&= sales \cdot (1 - EV).
\end{aligned} \qquad [2.17]
$$

The lottery controller's objective function becomes

$$
\max_p \pi = p \cdot q(p) - q(p) \quad , \qquad [2.18]
$$

where the first-order condition with respect to the effective price yields

$$
\begin{aligned}
& q + p\frac{\partial q}{\partial p} - \frac{\partial q}{\partial p} = 0 \\
& \Rightarrow \frac{\partial q}{\partial p}(p-1) = -q \\
& \Rightarrow \frac{\partial q}{\partial p}\frac{q}{q} = -\frac{q}{(p-1)}\frac{p}{q} \\
& \Rightarrow \eta_p^* = -\frac{p}{p-1} = -\frac{1}{1-EV}. \qquad [2.19]
\end{aligned}
$$

In this case, the profit-maximizing price elasticity becomes a function of the expected value of a bet, implying the elasticity that maximizes the lottery controller's objective function is different for every realized expected value.

This siutation leads to an interesting question of how to determine whether the lottery is optimally priced. As demonstrated in the previous subsection regarding the use of a mean-evaluated price elasticity, the data suggest that demand is not iso-elastic. Therefore the results of the static optimization problem indicate that in order to maximize profits, the elasticities of the empirical demand curve should equal the derived optimal elasticity at each individual effective price.

2.3 DATA

The data for this study were collected from public records provided by the Texas Lottery Commission (TLC) and include daily sales, advertised jackpots, and odds at each prize tier for the games Lotto Texas spanning April 2006 to year's end 2009. This information also was collected for the games Texas Two Step and Mega Millions, which are also offered by the TLC with effective prices that enter the model as a control for competing games. Summary statistics for Lotto Texas are provided in TABLE 2.1.

TABLE 2.1. Descriptive Statistics and Prize Information

N = 385	**Mean (μ)**	**St. Dev. (σ)**	**Min**	**Max**
Ticket Sales	2,106,588	491,836	1,377,250	4,208,545
Top Prize	17,220,779	14,533,925	4,000,000	76,000,000
Expected Value (*EV*)	0.48	0.33	0.17	1.86
Effective Price: $A = 1\text{-}EV$	0.52	0.67	-0.86	0.83
Effective Price: $p = 1/EV$	2.81	1.35	0.53	5.81
	1st Prize	2nd Prize	3rd Prize	4th Prize
Prize Amount	Jackpot*	$2000*	$50*	$3**
Odds	1:25,827,165	1:89,678	1:1,526	1:75

*Pari-mutuel prize, ** Guaranteed prize

Tickets for Lotto Texas are available for purchase every day of the week, but the actual drawings are only held on Wednesday and Saturday evenings. Sales per drawing range from 1.4 million to 4.2 million, with an average of approximately 2.1 million over the sample period. The base jackpot for Lotto Texas is $4 million reaching a high of $76 million, resulting from 47 drawings without a winner. The range of expected values spans the interval of $0.17 to $1.86, with a mean of $0.48. Thus in the long run, players can expect approximately $0.48 for every dollar they spend. Descriptive statistics for the two effective price definitions are given as well. Regarding the effective price from the lottery controller's perspective, A, the average profit per ticket sold is $0.52, and ranges from a high of $0.83 to a low of -$0.86. From the perspective of consumers, the effective price, p, ranges from $0.54 to $5.81, with a mean of $2.81. Again, this value represents the amount a player must spend, on average, in order to win a dollar.

Lotto Texas offers four prize tiers, each corresponding to whether all or some of the numbers are correctly matched. In each drawing six numbers are randomly obtained from a field of 54. The jackpot is won by matching all six at odds of 1:25,827,165. The 2nd and 3rd prizes are awarded from matching 5 and 4 balls, respectively. Since these prize tiers also are pari-mutuel, the amount a winner receives will depend on both the total draw sales as well as the number of co-winners. On average these values are $2,000 and $50, respectively. The 4th prize, for matching 3 numbers correctly, is guaranteed at $3, regardless of the number of co-winners.

TABLE 2.2. Log-Log Lotto Texas Regressions ($P - EV$ Approach)

Dependent Variable: Sales (in logs)
Independent effective price variables are each measured in logs

	OLS (actual sales)			2SLS			OLS (predicted sales)		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)
	Unaltered	Dropped	Censored	Unaltered	Dropped	Censored	Unaltered	Dropped	Censored
	n = 385	n = 345	n = 385	n = 385	n = 345	n = 385	n = 385	n = 345	n = 385
Price (Lotto Texas)	---	-0.1467	-0.1435	---	-0.2009	-0.1615	---	-0.1656	-0.1459
	---	0.000	0.000	---	0.000	0.000	---	0.000	0.000
Period	---	-0.0003	-0.0003	---	-0.0004	-0.0004	---	-0.0004	-0.0003
	---	0.000	0.000	---	0.000	0.000	---	0.000	0.000
Price (Mega Mil)	---	-0.0626	-0.0325	---	-0.0394	-0.0134	---	-0.0591	-0.0322
	---	0.000	0.129	---	0.022	0.351	---	0.002	0.141
Price (2 Step)	---	0.0049	-0.0086	---	0.0044	-0.0078	---	0.0045	-0.0086
	---	0.739	0.539	---	0.773	0.296	---	0.843	0.535
Constant	---	14.4873	14.4881	---	14.4730	14.4896	---	14.4804	14.4876
	---	0.000	0.000	---	0.000	0.000	---	0.000	0.000
Serial Correlation									
D-W Statistic	---	1.703	1.380	---	1.783	1.341	---	1.746	1.384
Q-stat (p-value)	---	0.000	0.000	---	0.000	0.000	---	0.000	0.000
Order (lags)	---	3	3	---	2	2	---	3	3

P-values are calculated using Newey-West standard errors and are reported under each coefficient estimate

TABLE 2.3. Log-Log Lotto Texas Regressions (*P/EV* Approach)

Dependent Variable: Effective quantity (in logs)
Independent effective price variables are each measured in logs

	OLS (actual sales)			2SLS			OLS (predicted sales)		
	(i) Unaltered n = 385	(ii) Dropped n = 345	(iii) Censored n = 385	(iv) Unaltered n = 385	(v) Dropped n = 345	(vi) Censored n = 385	(vii) Unaltered n = 385	(viii) Dropped n = 345	(ix) Censored n = 385
Price (Lotto Texas)	-1.3187	-1.2197	-1.4258	-1.3606	-1.2335	-1.6225	-1.3016	-1.2009	-1.4034
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Period	-0.0004	-0.0004	-0.0002	-0.0004	-0.0037	-0.0037	-0.0003	-0.0004	-0.0003
	0.000	0.000	0.175	0.000	0.000	0.000	0.000	0.000	0.178
Price (Mega Mil)	-0.0305	-0.0425	-0.0328	-0.0138	-0.0385	0.0393	-0.0295	-0.0422	-0.0323
	0.066	0.003	0.324	0.328	0.002	0.190	0.076	0.003	0.330
Price (2 Step)	-0.0203	-0.0086	0.0022	-0.0174	-0.0087	0.0136	-0.0205	-0.0093	0.0015
	0.255	0.690	0.955	0.354	0.641	0.741	0.254	0.662	0.970
Constant	14.9848	14.8799	15.0704	15.0109	14.8909	15.1985	14.9881	14.8811	15.0711
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Serial Correlation									
D-W Statistic	1.206	1.769	0.440	1.234	1.792	0.716	1.182	1.793	0.443
Q-stat (p-value)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Order (lags)	4	4	4	2	2	2	2	3	4

P-values are calculated using Newey-West standard errors and are reported under each coefficient estimate

TABLE 2.4. Semi-Log Lotto Texas Regressions ($P - EV$ Approach)

Dependent Variable: Sales (in logs)
Independent effective price variables are each measured absolutely

	OLS (actual sales)			2SLS			OLS (predicted sales)		
	(i) Unaltered n = 385	(ii) Dropped n = 345	(iii) Censored n = 385	(iv) Unaltered n = 385	(v) Dropped n = 345	(vi) Censored n = 385	(vii) Unaltered n = 385	(viii) Dropped n = 345	(ix) Censored n = 385
Price (Lotto Texas)	-0.5577	-0.5379	-0.7474	-0.5612	-0.5393	-0.8391	-0.5614	-0.5387	-0.7456
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Period	-0.0004	-0.0004	-0.0003	-0.0004	-0.0004	-0.0004	-0.0037	-0.0004	-0.0003
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Price (Mega Mil)	-0.0923	-0.0950	-0.0570	-0.0865	-0.0945	-0.0140	-0.0851	-0.0942	-0.0556
	0.001	0.008	0.127	0.001	0.001	0.672	0.005	0.008	0.127
Price (2 Step)	-0.0165	0.0021	0.0034	-0.0348	0.0021	0.0102	-0.0344	0.0021	-0.0019
	0.526	0.957	0.901	0.149	0.951	0.764	0.262	0.958	0.936
Constant	15.0108	14.9883	15.0915	15.0218	14.9889	15.1185	15.0236	14.9912	15.0962
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Price Elasticity (Lotto Texas)	-0.249	-0.239	-0.332	-0.249	-0.239	-0.373	-0.249	-0.239	-0.331
<u>Serial Correlation</u>									
D-W Statistic	1.898	1.985	1.269	1.899	1.987	1.260	1.906	1.990	1.285
Q-stat (p-value)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Order (lags)	3	3	3	2	2	2	3	3	2

P-values are calculated using Newey-West standard errors and are reported under each coefficient estimate

2.4 RESULTS

TABLES 2.2, 2.3, and 2.4 summarize the results of 9 different regression specifications under three EPM approaches: (1) log-log with $p = P - EV$, (2) log-log with $A = P/EV$, and (3) semi-log with $A = P - EV$, respectively. The columns are organized first according to the method by which endogeneity is addressed: OLS without controlling for endogeneity, two-stage least squares, and the direct use of predicted sales. Then the columns are sorted by the treatment of observations with super-unitary expected values: unaltered, omitted, and censored at \$0.99. The data were tested for serial correlation using a Ljung-Box test (Q statistic). These test statistics, along with the Durbin-Watson test statistics reported in the table. A correlogram was used for each regression to determine the appropriate number of lags to specify in the correction procedure. In order to control for the presence of serial correlation, Newey-West standard errors were estimated. The associated p-values are reported under each coefficient in the table.

In TABLE 2.2, it is not possible to compare the effects of the data treatments to the unaltered sample so the columns pertaining to the unaltered sample have been intentionally left blank. It is still possible to conclude that, given the censored dataset, the effect of endogeneity is still relatively small, which can be seen by comparing the price coefficients from columns (iii) and (ix), a difference of only 1.6%. In addition, the difference between the coefficient estimates of the omitted- and censored-sample specifications is relatively large under both 2SLS and predicted sales methods, suggesting the impact of the observations with super-unitary expected values is likely substantial in the *P–EV* model, although it is not directly testable in this framework.

The results for the log-log model with $p = P/EV$ are presented in TABLE 2.3. In this model, the effective price coefficient estimates for Lotto Texas are interpreted as the price elasticity of an iso-elastic demand curve and are statistically significant at the 1% level in each of the regressions. Comparison of the Lotto Texas price coefficient under column (i) with that under columns (iv) and (vi) shows the difference in the results from the two methods for controlling for endogeneity under the full, uncensored sample. Under two-stage least squares, the price coefficient is estimated to be -1.3606, which is 3.2% larger in magnitude then uncontrolled estimate of -1.3187. Using predicted sales to control for endogeneity results in a coefficient estimate of -1.3016, which is 1.3% smaller in magnitude than the uncontrolled estimate. Both of these differences are statistically significant at the 1% level.

According to the data, the effect of censoring or dropping the observations with super-unitary expected values is much larger than that of failing to control for endogeneity. The last three columns (vii – ix) show the results using the predicted sales method. The effective price coefficient under the unaltered dataset is estimated to be -1.3016. Omitting the super-unitary observations results in a coefficient estimate of -1.2009 (a 7.7% decrease in magnitude against the unaltered estimated), while censoring the observations results in a coefficient estimate of -1.4034 (a 7.8% increase in magnitude against the unaltered estimate). These differences are also statistically significant at the 1% level. Under 2SLS the effects of omitting and censoring the data are even larger. Under the unaltered sample, the coefficient estimate is -1.3606, which leads to a 9.2% decrease in magnitude if the observations are dropped (-1.2335) and a 19.2% increase in magnitude if the observations are censored (-1.6225). Clearly, the choice of how these observations are treated has a large impact on the results of the log-log *P/EV* model.

The results for the semi-log approach are given in TABLE 2.4. In this case the bias incurred from censoring (compared to the unaltered sample) is even larger, yielding differences of 14.4%, 20.4%, and 14.2% for the OLS_{act}, 2SLS, and OLS_{pred} models, respectively. However, simply dropping the extreme observations only yields differences of 1.5%, 1.6%, and 1.7%, for the OLS_{act}, 2SLS, and OLS_{pred} models, respectively, when compared to the unaltered samples. With the difference in the coefficients resulting from endogeneity amounting to only 0.3%, the semi-log model also provides compelling evidence the treatment of extreme observations is of more serious concern than endogeneity.

While the p-values reported for the Lotto Texas price coefficients in the three tables correspond to a statistical test against zero, it is more important to test whether the price elasticities are statistically different from their respective profit maximizing level. Expressions [2.16] and [2.19] provide the static profit maximization conditions for the producer and consumer approaches, respectively. Expression [2.16] requires the elasticity of a model using $A = P - EV$ to be -1 in order to be profit maximizing. According to the semi-log model under the unaltered sample using predicted sales to control for endogeneity (TABLE 2.4, column (ix)), the estimated price elasticity of

-0.331 is statistically different from -1 at the 1% level with a p-value of 0.000. Similarly, expression [2.19] requires the elasticity of a model using $p = P/EV$ to be -2.272. This is derived by using the sales-weighted sample-mean expected value, $\overline{EV} =$ 0.560. According to the log-log model under the unaltered sample using predicted sales to control for endogeneity (TABLE 2.3, column (ix)), the estimated price elasticity of -1.4034 is statistically different from -2.272 at the 1% level with a p-value of 0.000. Both of these results indicate that the price of Lotto Texas is not optimal and should be raised to increase profits.

However, the results of both of these tests for profit maximization are based upon estimates that are evaluated at the sample mean and are not necessarily valid at other points along the demand curve. Therefore it may be necessary to evaluate the profit maximization condition at each point along the demand curve. This can be done using the LOWESS regression analysis. Under the effective price definition of $P - EV$, the estimated arc elasticities obtained using LOWESS are plotted in FIGURE 2.6. The black dots denote the LOWESS smoothed arc elasticities computed at each point along the demand curve. It is clear that the curve is everywhere inelastic relative to the profit maximizing value of -1. FIGURE 2.7 provides a similar result when using the effective price definition of *P/EV*. In this case the optimal elasticity varies along the demand curve as dictated by expression [2.19]. The LOWESS smoothed arc elasticities (denoted as black dots) are everywhere inelastic relative to the optimum.

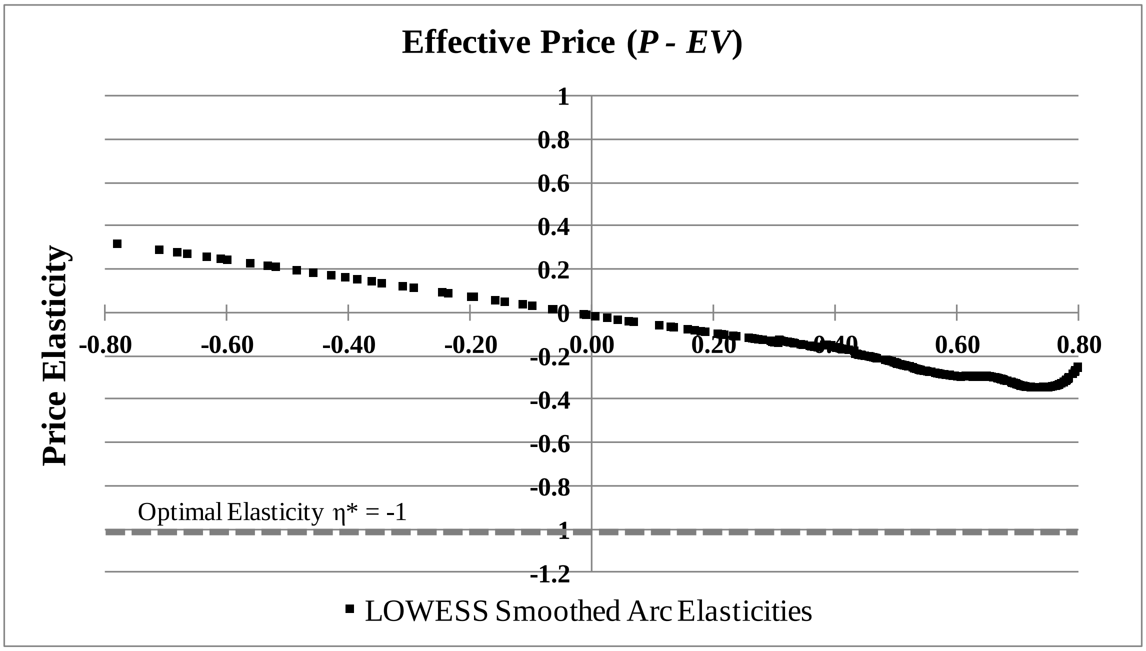

FIGURE 2.6. Lotto Texas Arc Elasticities vs. Optimal Elasticities Derived from the Lottery Controller's Static Profit Maximization Problem (*A* = *P* – *EV*)

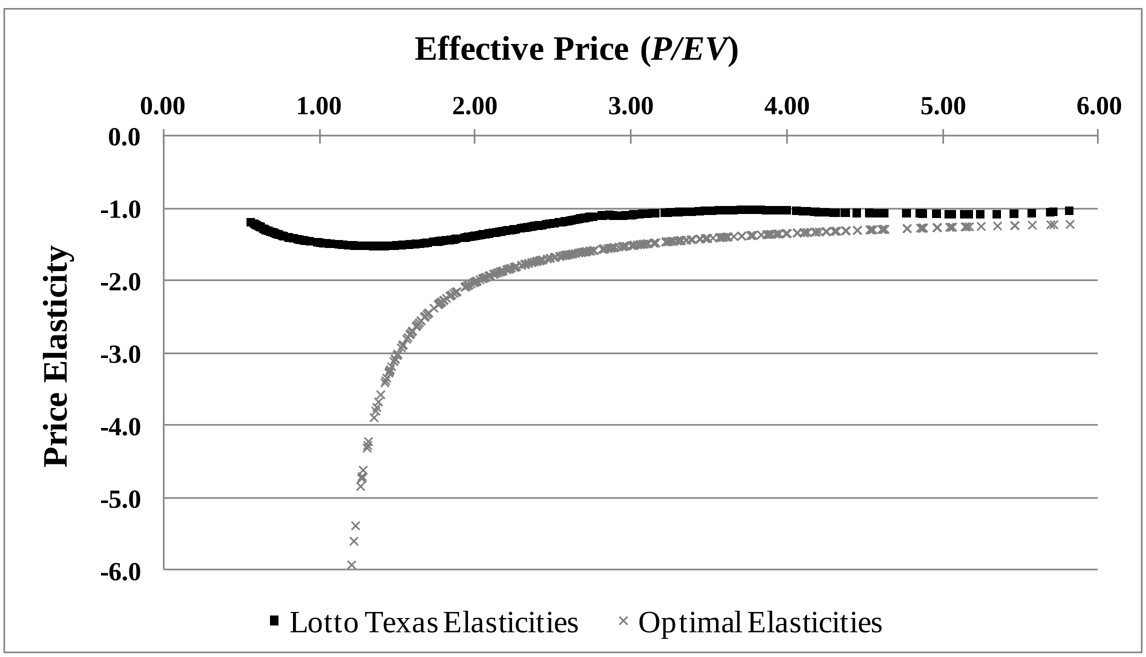

FIGURE 2.7. Lotto Texas Arc Elasticities vs. Optimal Elasticities Derived from the Lottery Controller's Static Profit Maximization Problem (*p* = *P/EV*)

2.5 CONCLUSIONS

The aim of this essay was to re-examine the effective price model of lottery demand within a new framework that addresses several outstanding issues in the literature. The predominant approach to modeling the demand for lottery tickets revolves around the use of an econometric framework where sales are regressed on a game's effective price. Estimation of this model in logarithms appears to provide a convenient way to obtain price elasticities, however the precise estimates are not entirely robust to either the definition of the effective price or the regression model specification.

I have shown that the definition of $A = P–EV$ is problematic given this framework due to rolling jackpots that can cause the expected value of a ticket to exceed the nominal value, requiring omission or censoring of the data if to still be estimated in logs. I addressed this issue in two ways: (1) estimation of the price elasticity in a semi-log framework, and (2) by proposing a new definition of the effective price, $p = P/EV$, that that is compatible under logarithmic transformations. I also addressed the issue of the endogeneity of price using a unique set of sales predictions that enter into the effective price calculation directly, circumventing the need to estimate a two-stage model. Finally, examine the use of a non-parametric regression technique to obtain estimates of a lottery game's price elasticities along the entire demand curve. The estimated price elasticities were then used to test whether Lotto Texas has effectively maximized profits.

The results from the various modeling specifications provide compelling evidence that way observations with super-unitary expected values are treated in the model can have a substantial impact on the estimated results. In fact, according to the Lott Texas data, the bias incurred from failing to properly incorporate the extreme-jackpot observations was six times the size of the bias incurred by failing to control for endogeneity. Using a log-log model imposes an iso-elastic curvature restriction on the data. Relaxing this restriction and estimating demand using local linear regression revealed that the demand curve is not iso-elastic, lending doubt to the validity of using elasticities evaluated at the sample mean to summarize price sensitivity over the entire demand curve. Ultimately, the results from each of the demand models specified in this section indicated that Lotto Texas is not profit maximizing and can benefit substantially from a price increase.

The methods described in this section exhibit a number of limitations. First, this analysis investigates the demand behavior for a single game in isolation. It is reasonable to suspect that price changes for competing products could have a considerable impact on the demand for Lotto Texas. Since Texas operates multiple games concurrently within its portfolio, this situation would provide a logical extension to the analysis. This particular issue will be explored in further detail in the following section. Second, since the jackpots of lotto games are related intertemporally, static analysis of profitability will not effectively control for the effect a rollover may have on expected profits of future draws. This topic will be addressed in Section 4. Third, this study did not control for the potential effects of addictive behavior in relation to lottery gambling. This topic has been addressed to some extent in the literature and it may be valuable to investigate the role of addiction within the context of the models presented in this section.

3. DEMAND FOR LOTTERY GAMBLING: THE USE OF A DIFFERENTIAL DEMAND SYSTEM TO EVALUATE PRICE SENSITIVITY WITHIN A PORTFOLIO OF LOTTERY GAMES

3.1 INTRODUCTION

In this essay, I build upon the methods described in Section 2 by extending the analysis to model lottery demand over an entire portfolio of lottery products. It is rare to observe a lottery controller that operates a single game in isolation, motivating the need to examine, from a profitability perspective, how players respond to relative changes in the betting values of competing games.

This analysis is important for three main reasons. First, the ability to identify the degree to which games within a common portfolio are economic substitutes or complements of each other can help operators better market their products to consumers. For instance, if two games are estimated to be economic complements, then there may be potential gains in promoting one at the time of purchase of the other. Second, assuming individuals have a fixed budget for lottery spending, then there arises the possibility of sales cannibalization among games that compete for a player's dollar. Thus profit analysis must be conducted over the entire portfolio, maximizing the total amount of profit generated over all games, instead of maximizing the profit generated by each game individually. Third, current methods in demand system analysis can provide both compensated and uncompensated price elasticities, expenditure elasticities, and provides identification of demand parameters for games that exhibit no variation in the effective price.

The price elasticities estimated using the demand system can be used to measure the profitability of entire lottery portfolio. The conditions for profit maximization are obtained by solving the lottery controllers objective function which subtracts total lottery payouts from total revenue earned from ticket sales.

3.2 METHODOLOGY

3.2.1 Barten's Synthetic Demand System

In this study, own-price, cross-price, and expenditure elasticities are estimated using Barten's (1993) synthetic differential demand system, which is a generalization of four commonly used specifications in demand analysis: the Rotterdam model (Barten 1964; Theil 1965; Barnett 1979; and Mountain 1988), the first-differenced Linear Approximate Almost Ideal Demand System (Deaton and Muellbauer 1980), the (Dutch) Central Bureau of Statistics model (Keller and van Driel 1985), and the National Bureau of Research model (Neves 1987). Barten's synthesis combines the flexible functional form of the AIDS model with the econometric simplification of the differential systems. The model is both linear in its parameters (making it relatively easy to estimate) and allows for the first-differencing of variables to address issues of non-stationarity (Matsuda 2005).

To derive Barten's demand system with respect to the games' effective prices, I follow the notation in Matsuda (2005). Let m be total expenditure on n games and let

$p = (p_1,\ldots, p_n)$ be a vector of effective prices defined according to expression [2.3] in the previous section. Again, by defining the effective price in this manner, it allows the model to include observations with super-unitary expected values since Barten's demand system also relies upon a logarithmic transformation of the games' effective prices. Totally differentiating the effective Marshallian demand, $q_i(p,m)$, yields

$$dq_i(p,m) = \frac{\partial q_i(p,m)}{\partial m} dm + \sum_{j=1}^{n} \frac{\partial q_i(p,m)}{\partial p_j} dp_j, \quad i = 1,\ldots,n. \qquad [3.1]$$

By the Slutsky equation we get,

$$\frac{\partial q_i(p,m)}{\partial p_j} = \frac{\partial h_i(p,u)}{\partial p_j} - \frac{\partial q_i(p,m)}{\partial m} q_j(p,m) \qquad [3.2]$$

where $h_i(p,m)$ represents the Hicksian demand of game i. Finally, the budget constraint (adding-up condition), $m = \sum_{i=1}^{n} p_i q_i$, is differentiated and algebraically rearranged to form

$$\sum_{i=1}^{n} p_i dq_i = dm - \sum_{i=1}^{n} q_i dp_i \,. \qquad [3.3]$$

By substituting [3.2] into [3.1] with the aid of [3.3], then multiplying both sides by p_i/m we get

$$w_i d\log q_i = p_i \frac{\partial q_i}{\partial m} d\log \tilde{Q} + \sum_{j=1}^{n} \frac{p_i p_j}{m} \frac{\partial h_i}{\partial p_j} d\log p_j \,. \qquad [3.4]$$

On the right-hand side of this equation,

$w_i d\log q_i$ represents the log-change in effective quantity of game i, weighted by its respective expenditure share, $w_i \equiv \dfrac{p_i q_i}{m}$.

On the left-hand side:

$p_i \dfrac{\partial q_i}{\partial m}$ denotes the marginal expenditure share of game *i* and determines how additional expenditure to this game is allocated;

$d \log \tilde{Q} = \sum_i w_i d \log q_i$ denotes the Divisia volume index number for changes in real income. Note the use of the tilde on $\tilde{Q}$ is intended to differentiate this term from that used to denote the total tickets sales quantity, *Q*; and

$\dfrac{p_i p_j}{m} \dfrac{\partial h_i}{\partial p_j}$ denotes the ij^{th} element of the Slutsky matrix and pertains to the substitution effect between games *i* and *j* in response to a change in game *j*'s effective price (represented by $d \log p_j$).

In Barten's model, $p_i \dfrac{\partial q_i}{\partial m}$ is assumed to equal $(\alpha_i + \delta_1 w_i)$ and $\dfrac{p_i p_j}{m} \dfrac{\partial h_i}{\partial p_j}$ equals $\beta_{ij} - \delta_2 w_i (\delta_{ij} - w_j)$, resulting in the general form:

$$w_i d \log q_i = (\alpha_i + \delta_1 w_i) d \log Q + \sum_{j=1}^{n} \left[\beta_{ij} - \delta_2 w_i (\delta_{ij} - w_j) \right] d \log p_j + \mu_i \qquad [3.5]$$

where $\alpha_i \equiv (1 - \delta_1) b_i + \delta_1 c_i$, is a vector of expenditure coefficients, $\beta_{ij} \equiv (1 - \delta_2) s_{ij} + \delta_2 r_{ij}$ is a matrix of price coefficients, and μ_i is a stochastic error term. The estimated values for the parameters δ_1 and δ_2 allow the general model to be decomposed into the four aforementioned demand system specifications according TABLE 3.1.

TABLE 3.1. Barten Decomposition Parameters

Model	Parameter	
	δ1	δ2
Rotterdam	0	0
Linear Approx. Almost Ideal Demand System (LA-AIDS)	1	1
Central Bureau of Statistics (CBS)	1	0
National Bureau of Research (NBR)	0	1

For $\delta_1 = 0$ and $\delta_2 = 0$, the general model reduces to the Rotterdam model, or

$$w_i d \log q_i = b_i d \log \tilde{Q} + \sum_{j=1}^{n} s_{ij} d \log p_j \ , \qquad [3.6]$$

which is one of the first and most commonly used differential demand systems. In this model, both the marginal expenditure share b_i and the matrix of Stutsky parameters s_{ij} for each game are both assumed constant. Although the model is relatively simple to estimate these assumptions are highly restrictive. For $\delta_1 = 1$ and $\delta_2 = 0$, we get the Central Bureau of Statistics (CBS) model,

$$w_i d \log q_i = \left(c_i + w_i\right) d \log \tilde{Q} + \sum_{j=1}^{n} s_{ij} d \log p_j \ , \qquad [3.7]$$

which relaxes the restriction of constancy for the marginal expenditure shares by defining $c_i \equiv b_i - w_i$ and substituting for b_i. Conversely, relaxing the constancy restriction on the Slutsky parameters s_{ij} yields the National Bureau of Research (NBR) model

$$w_i d \log q_i = b_i d \log \tilde{Q} + \sum_{j=1}^{n} \left[r_{ij} - w_i \left(\delta_{ij} - w_j \right) \right] d \log p_j \ , \qquad [3.8]$$

where $r_{ij} \equiv s_{ij} + w_i\left(\delta_{ij} - w_j\right)$ with δ_{ij} denoting Kronecker's delta, which is equal to 1 if *i*=*j* and 0 if *i*≠*j*. This specification is obtained from [3.5] when $\delta_1 = 0$ and $\delta_2 = 1$. Relaxing both assumptions yields the differential Linear Approximate Almost Ideal Demand System (LA-AIDS) model

$$w_i d\log q_i = \left(c_i + w_i\right) d\log\tilde{Q} + \sum_{j=1}^{n}\left[r_{ij} - w_i\left(\delta_{ij} - w_j\right)\right] d\log p_j \ . \qquad [3.9]$$

The AIDS was model originally developed by Deaton and Muellbauer (1980) and uses a flexible functional form, which acts as a second-order approximation to the true indirect utility function. While the AIDS specification is non-linear in its parameters (arising from its use of a translog price index), the "linearized approximate" AIDS model uses Stone's (1953) linear price index to approximate the translog index. Expression [3.9] provides a first-differenced linear approximate AIDS specification that is obtained from [3.5] when $\delta_1 = 1$ and $\delta_2 = 1$. It is important to note that estimation using Barten's synthesis does not impose any restrictions on the nesting parameters δ_1 and δ_2. Consequently, it is not necessary for the general model to fully reduce at all and can therefore be utilized as a demand system in and of itself (Brown 1994). However, as pointed out by Matsuda (2005), the economic implications of δ_1 & δ_2 outside the interval [0,1] seem to be unclear.

The familiar demand restrictions for the general Barten model are given by

$$\sum_i \beta_{ij} = 0, \sum_i \alpha_i = 1 - \delta_1 \quad \text{(Adding-up)}, \tag{3.10}$$

$$\beta_{ij} = \beta_{ji} \quad \text{(Symmetry), \&} \tag{3.11}$$

$$\sum_j \beta_{ij} = 0 \quad \text{(Homogeneity).} \tag{3.12}$$

These restrictions provide the model with consistency to economic theory and assist with parameter identification. The procedure for estimating [3.5] requires the omission of a single equation, but the parameters can be recovered by imposing the demand restrictions. This method is convenient for identifying the parameters for a game that exhibits no effective price variation since its corresponding equation can be the one omitted from the system.

The expenditure elasticity of demand with respect to game *i* is given by

$$e_i = \frac{\alpha_i + \delta_1 w_i}{w_i} \tag{3.13}$$

and measures the responsiveness of demand to changes in expenditure on lottery games in general within the sample. The uncompensated are compensated elasticities are

$$\eta_{ij} = -\left(\frac{\alpha_i + \delta_1 w_i}{w_i}\right) w_j + \frac{\beta_{ij} - \delta_2 w_i(\delta_{ij} - w_j)}{w_i}, \tag{3.14}$$

$$\tilde{\eta}_{ij} = \frac{\beta_{ij}}{w_i} - \delta_2 \left(\delta_{ij} - w_j\right) \tag{3.15}$$

respectively.

3.2.2 Two-Stage Budgeting for Lottery Games

In reality, individuals' lottery consumption patterns are inextricably linked to countless external factors such as the prices of all other consumable goods. A complete analysis would be impossible; therefore it becomes necessary to impose simplifying assumptions on consumer preferences, particularly regarding how individuals allocate their budget for each good they purchase. Edgerton (1997) describes the implications of two-stage budgeting on the estimation of price and expenditure elasticities for different groups of goods. Under an assumption of two-stage budgeting, an individual's allocation decision takes place in two independent steps. First, total expenditure is allocated between *m* broad groups of goods. Second, the expenditure on each group is then allocated among *n* elementary goods within each group (e.g. the first-stage budgeted amount for lottery gambling is then distributed among various games: Lotto Texas, Mega Millions, etc.).

A necessary and sufficient condition for the second stage is that of weak separability of utility. According to Pollock and Wales (1992), by assuming weak-separability, the demand for a single lottery game can be expressed as a function of the prices of all the games within the lottery group and total expenditure on lottery as a whole. This implies that expenditure and the prices of goods outside of the lottery group (i.e. food or clothing) enter the demand functions for lottery games only through their effect on total expenditure the lottery group. Thus, since this analysis only deals with expenditure on lottery rather than total expenditure on all goods, the prices of all non-lottery goods can be ignored, which mitigates the complexity of the problem. The assumption of weak separability is not unreasonably restrictive since one would not expect a change in the price of non-lottery goods to affect the relative marginal rates of substitution among various games within the lottery group.

As discussed in Edgerton (1997), two-stage budgeting has important implications on the estimated price elasticities within the lottery group. A change in the price of one game will have a direct effect on the quantities purchased within the lottery group as well as an indirect effect on the allocation of total lottery-group expenditure. The effect of a change in the quantity of game *i* resulting from a change in the effective price of game *j* under two-stage budgeting is expressed mathematically as:

$$\frac{\partial \ln q_i}{\partial \ln p_j} = \frac{\partial \ln h_i}{\partial \ln p_j} + \frac{\partial \ln h_i}{\partial \ln X} \cdot \frac{\partial \ln X}{\partial \ln p_j}, \qquad [3.16]$$

where h_{i} is the Marshallian demand for game *i*, and *X* is total expenditure on the lottery group. The term ln*X* can be decomposed into

$$\ln X = \ln Q^* + \ln P^*, \qquad [3.17]$$

where Q^* represents aggregate demand and P^* is the price index for the lottery group. Combining [3.17] with Stone's (1953) price index,

$$\ln P^* = \sum_{n=1}^{k} w_k \ln p_k \ , \qquad [3.18]$$

expression [3.16] becomes

$$\frac{\partial \ln q_i}{\partial \ln p_j} = \frac{\partial \ln h_i}{\partial \ln p_j} + \frac{\partial \ln h_i}{\partial \ln X}\left(\frac{\partial \ln P^*}{\partial \ln p_j} + \frac{\partial \ln Q^*}{\partial \ln P^*} \cdot \frac{\partial \ln P^*}{\partial \ln p_j} \right). \qquad [3.19]$$

Equation [3.19] can be expressed in elasticity form according to

$$\xi_{ij} = \eta_{ij} + e_i w_i \left(1 + \phi\right), \qquad [3.20]$$

where η_{ij} is the within-group uncompensated price elasticity of game i with respect to game j, e_i is the within-group expenditure elasticity for game i, w_i is the budget share for game i, and φ is the uncompensated price elasticity of aggregate demand with respect to the price index. To summarize, ξ_{ij} represents the uncompensated effective price elasticity adjusted to account for two-stage budgeting. It is important to note that given the available data, obtaining the corresponding adjusted expenditure and compensated price elasticities is impossible since it would require total expenditure on all groups, which is unavailable. Therefore only the adjusted uncompensated price elasticities are reported.

3.2.3 Profit Maximization over a Portfolio of Games

Analysis of a system of games requires the lottery controller's objective function (see expression [2.18] in the previous section) to be summed over all games. Thus the objective function over the portfolio becomes:

$$\max_{p_i} \pi = \sum_{i=1}^{N} \left[p_i q_i \left(p_i \right) - q_i \left(p_i \right) \right] \ \forall i \in N \, . \tag{3.21}$$

Taking the first-order conditions and simplifying yields:

$$i: q_i + \sum_{j=1}^{N} p_j \frac{\partial q_j}{\partial p_i} - \sum_{j=1}^{N} \frac{\partial q_j}{\partial p_i} = 0$$

$$\Rightarrow q_i + \sum_{j=1}^{N} \left(p_j - 1 \right) \frac{\partial q_j}{\partial p_i} = 0$$

$$\Rightarrow \sum_{j=1}^{N} \frac{\left(p_j - 1 \right)}{q_i} \frac{\partial q_j}{\partial p_i} = -1$$

$$\Rightarrow \sum_{j=1}^{N} \frac{\left(p_j - 1\right)}{q_i} \frac{p_i}{p_i} \frac{q_j}{q_j} \frac{\partial q_j}{\partial p_i} = -1$$

$$\Rightarrow \sum_{j=1}^{N} \frac{\left(p_j - 1\right)}{q_i} \frac{q_j}{p_i} \left(\frac{\partial q_j}{\partial p_i} \frac{p_i}{q_j} \right) = -1$$

$$\Rightarrow \sum_{j=1}^{N} \frac{\left(p_j - 1\right)}{q_i} \frac{q_j}{p_i} \xi_{ji} = -1$$

$$\Rightarrow \sum_{j=1}^{N} \left(p_j q_j - q_j \right) \xi_{ji} = -p_i q_i$$

$$\Rightarrow \sum_{j=1}^{N} \left(\frac{p_j q_j}{x} - \frac{q_j}{x} \right) \xi_{ji} = -\frac{p_i q_i}{x}$$

$$\Rightarrow \sum_{j=1}^{N} \left(w_j - \frac{q_j}{x} \right) \xi_{ji} = -w_i$$

Re-indexing the above expression we get

$$\sum_{i=1}^{N} \left(w_i - \frac{q_i}{x} \right) \xi_{ij} = -w_j$$

$$\Rightarrow \sum_{j=1}^{N} \sum_{i=1}^{N} \left(w_i - \frac{q_i}{x} \right) \xi_{ij} = -1, \qquad [3.22]$$

where x is total expenditure on all games, w_i is the expenditure share for game i, and ξ_{ij} is the Marshallian price elasticity of demand for game i with respect to game j. This result indicates that the optimal profit depends not only upon the individual price elasticities of each game, but also their related cross-price elasticities and total expenditure. Furthermore, estimation of a point elasticity (e.g. evaluated at the mean) will require specification of mean measures of both the expenditure shares and the effective quantities of each game. An important implication of expression [3.22] is that profit maximization for the portfolio is not necessarily achieved when the own-price elasticity of each individual game is equal to -1.

3.3 DATA

Texas is an ideal candidate for studying lottery demand for a number of reasons. First, Texas has relatively few gaming substitutes to the lottery, making it an isolated market where the sales of lottery tickets are unlikely to be influenced by other gambling activities. With the exception of six scattered racetracks and a lone Native American casino located in Eagle Pass at the border of Mexico, the lottery is virtually the only source of gambling available to public within the state of Texas[3]. Second, the vast majority of the population in the state is located on the interior, making the day-to-day cross-border shopping of lottery tickets a negligible issue. Third, the variety of available on-line lottery games offered by the Texas Lottery lends itself aptly to the study of demand among games in portfolio.

The data for this study were collected from public records provided by the Texas Lottery Commission and include daily sales, advertised jackpots, and odds at each prize tier for each of six games spanning April 2006 to year's end 2009. These games include Lotto Texas, Mega Millions, Texas Two Step, Cash Five, Pick 3, and Daily 4. Each game offers a different bundle of characteristics, creating a portfolio that appears to be tailored to suit a wide array of risk tolerance among players. These characteristics are

[3] I acknowledge the fact that Texas' neighboring states of Louisiana, Oklahoma, and New Mexico each have several gaming establishments that likely draw patronage of Texas residents. However, I assume that expenditure on lottery is budgeted separately from that of out-of-state gambling and that market dynamics between the two are independent.

summarized using descriptive statistics in TABLE 3.2. The number of drawings per week varies by game from twice a week to twice a day. In order to properly compare data among the six games, sales were aggregated to the level of the least frequently drawn games (i.e. twice a week). Aggregation in this manner allows each draw period to include every game at least once, and splits the number of drawings of the more frequent games (such as Cash Five, Pick 3, and Daily) in half. The actual draw dates are provided at the bottom of TABLE 3.2 and all the values in table are based on a half-week draw period basis.

All of the games in the sample were managed exclusively by the TLC with the exception of Mega Millions which was managed by a consortium of 12 states. Total sales for each participating state were gathered over the time period in order to obtain Mega Million's effective price. Predicted sales data were not available for Mega Millions, but it is reasonable to assume that Texas players have a negligible impact on the total level of sales, and thus the magnitude of simultaneity bias is likely to be inconsequential. Multi-state participation allows Mega Millions to offer jackpots at a much larger scale than any single-state lotto game, averaging \$66 million over both time periods at an odds ratio of 1:175,711,536. The long odds make Mega Millions the least generous of the games returning only 37 cents on average for every dollar bet. Despite offering a jackpot that is on average 3.6 times higher than that of Lotto Texas, the two game's average sales are very close in magnitude. Lotto Texas is the flagship game for the TLC offering jackpots starting at \$4 million and reaching up to \$76 million.

TABLE 3.2. Descriptive Statistics for the Texas Lottery Portfolio

N = 385	Mean (μ)	St. Dev. (σ)	Min	Max
Ticket Sales				
Lotto Texas	2,106,588	491,836	1,377,250	4,208,545
Mega Millions	2,206,366	1,467,058	1,134,096	16,500,000
Texas Two Step	472,937	150,316	293,085	1,159,439
Cash Five	250,323	42,895	134,978	380,351
Pick 3	2,861,647	213,066	1,888,025	3,491,319
Daily 4	527,649	94,656	331,838	1,158,071
Top Prize				
Lotto Texas	17,220,779	14,533,925	4,000,000	76,000,000
Mega Millions	62,130,208	57,809,074	12,000,000	370,000,000
Texas Two Step	422,987	373,679	200,000	2,900,000
Cash Five	27,459	3,286	17,217	36,261
Pick 3	500	0	500	500
Daily 4	5,000	0	5,000	5,000
Expected Value (EV)				
Lotto Texas	0.48	0.33	0.17	1.86
Mega Millions	0.37	0.15	0.22	0.96
Texas Two Step	0.46	0.16	0.36	1.50
Cash Five	0.41	0.004	0.40	0.42
Pick 3	0.50	0.00	0.50	0.50
Daily 4	0.50	0.00	0.50	0.50
Effective Price (P/EV)				
Lotto Texas	2.81	1.35	0.54	5.81
Mega Millions	3.13	1.03	1.05	4.59
Texas Two Step	2.34	0.51	0.69	2..81
Cash Five	2.43	0.03	2.36	2.50
Pick 3	2.00	0.00	2.00	2.00
Daily 4	2.00	0.00	2.00	2.00

	Odds	Draw Dates
Lotto Texas	1:25,827,165	Wed & Sat
Mega Millions	1:175,711,536	Tues & Fri
Texas Two Step	1:1,832,600	Mon & Thu
Cash Five	1:435,897	Mon-Sat
Pick 3	1:1,000	Twice Daily (except Sunday)
Daily 4	1:5,000	Twice Daily (except Sunday)

Sales per drawing fall within the range of 1.37 million to 4.2 million over the sample period with an average of 2.1 million. Lotto Texas has the highest variation in effective price among all the other games ranging from $.54 to $5.82. Texas Two Step is structured very similarly to Lotto Texas, though at a smaller scale with jackpots ranging from $200,000 to $2.9 million at odds of 1:1,832,600. Even though it offers more favorable odds, its lower jackpots do not appear to appeal as well to the public, generating sales of only about a fifth in magnitude to Lotto Texas.

Lotto Texas, Mega Millions, and Texas Two Step are designed according to the traditional lotto format with base jackpots that grow as rollovers accrue. The remaining three games follow different formats. In Cash Five, players choose five numbers from a field of 37 with prizes awarded for matching 2, 3, 4, or 5 numbers correctly, with drawings occurring once a day except on Sundays. In order to determine the value of each prize, all of lowest tier prizes ($2 for matching two numbers correctly) are paid out first, since this is a guaranteed prize. Once the total amount of $2 prizes has been subtracted from the prize pool, the residual is then allocated among the remaining prize tiers, with 40.15%, 18.08%, and 41.77% going to the first-, second-, and third-tier prizes, respectively. These prizes are then split among all winners in a pari-mutuel fashion. On average, the top prize amounts to about $27,500 at odds of 1:435,897. The mean effective price in the sample is 2.43, making Cash 5 cheaper to play than Texas Lotto or Mega Millions, on average.

Pick 3 is a small-scale game offering a top prize of only $500 with no rollovers. In Pick 3 a player chooses three single-digit numbers from 0 to 9 where bets are placed on the particular permutation of numbers chosen. For instance, one may play their numbers in exact order, any order, or both. All the prizes are fixed and computed in direct proportion to the odds of matching the chosen numbers. Pick 3 is the most popular on-line game in the portfolio boasting sales of 2 million tickets per drawing, on average, with drawings occurring twice a day (morning and night). It also is the least expensive game in the portfolio to play, at a fixed effective price of $2.00.

Added to the portfolio on October 1, 2007, Daily 4 is almost identical to Pick 3 except that players choose four single-digits from 0 to 9. This addition increases the scale of the game (in terms of odds and prize amounts) by ten and provides players with more available permutations by which to play their numbers. It appears that the lottery operator's intent of adding Daily 4 to the portfolio was to bridge the risk gap between Pick 3 and Cash Five, creating a game that only differs from Pick 3 by it scale. In its first month, sales of Daily 4 surged above those of Texas Two Step and Cash Five, but quickly fell to an equilibrium level of around 520,000 per draw period. Compared to Pick 3, Daily 4's poor ticket sales is not surprising considering that its design was likely tailored to a niche group of players with a slightly higher degree of risk tolerance.

In FIGURE 3.1, total sales of each of the six games by drawing are plotted. This figure clearly illustrates the relative magnitudes and variability of sales among the games. Texas lotto tends to perform better than Mega Millions in any given draw period, except where the Mega Millions jackpot is relatively high. In these drawings the sales of Mega Millions tickets rise very sharply as depicted by the tall peaks. Sales for Cash 5 and Daily 4 are relatively flat.

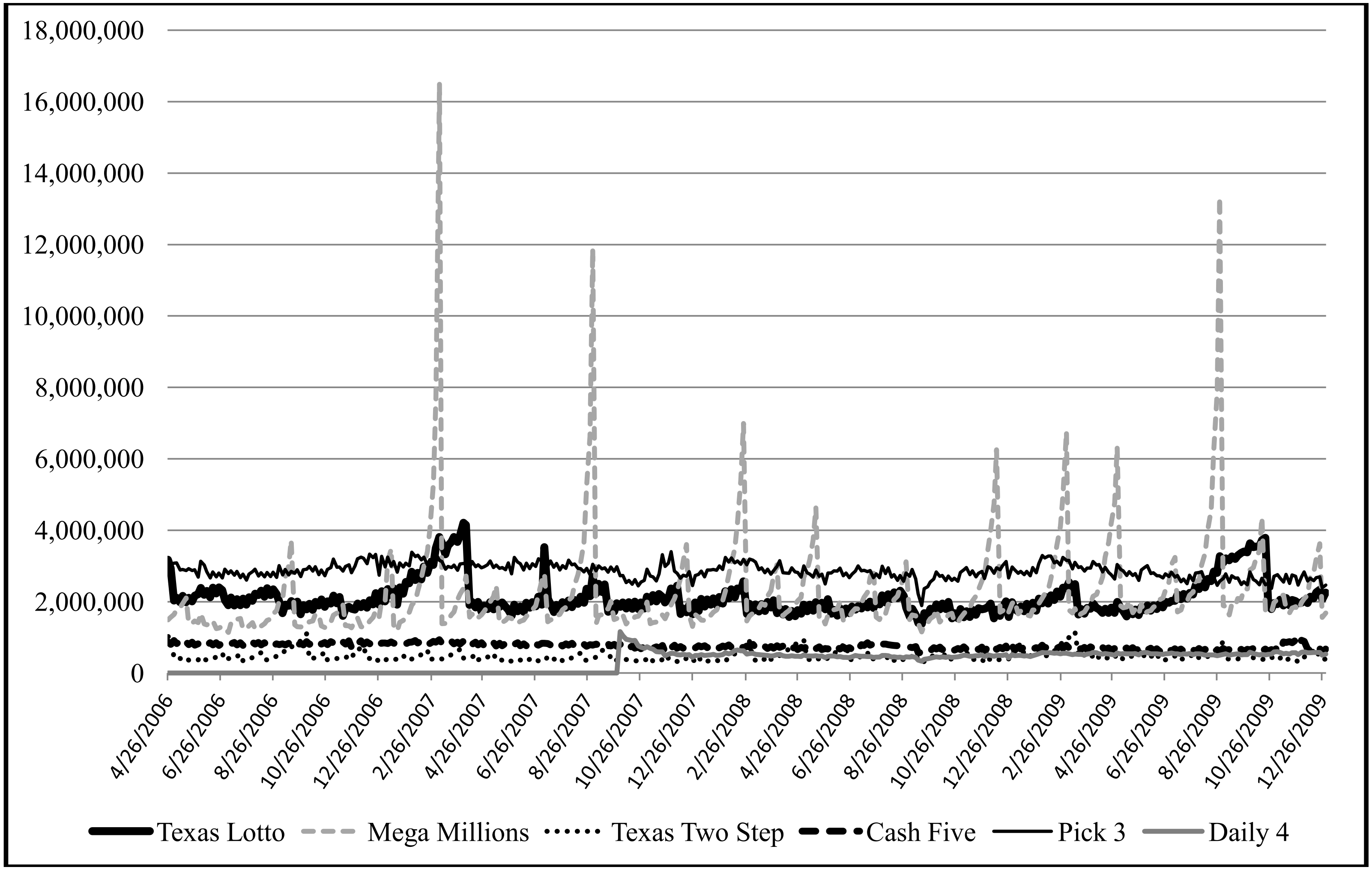

FIGURE 3.1. Texas Lottery Ticket Sales by Game

The effective prices of Pick 3 and Daily 4 do not vary. In order to identify the parameters used for computing the elasticities of these two games, I take advantage of the parameter restrictions of the Barten demand system (i.e. adding-up, homogeneity, and symmetry). Due to the fact that both of the games exhibit no price variation, it would be impossible to identify their parameters separately. As a result, I have chosen to lump them together and enter them into the model in the form of a composite game. However, their extreme similarity in design makes analyzing them as a composite a reasonable approach.

3.4 RESULTS

Using a differential demand system model to analyze lottery demand is advantageous in a number of ways. First, the model's demand parameters are easily combined to provide estimates of expenditure and price elasticities (both compensated and uncompensated) at any point along the demand curve. Second, using a formal demand system can provide results that are consistent with utility maximization. Third, the model controls for changes in expenditure levels on games in response to the indirect effects of price changes on the aggregate price index. Fourth, imposing demand restrictions can be used to identify games with little to no price variation. Fifth, the process of first-differencing variables in the model is likely to make them stationary. The model was estimated using a non-linear Seemingly Unrelated Regression procedure available in the STATA statistical software package.

In order to control for first-order serial correlation in the data, the demand system can be estimated in the form of an AR(1) process (Yuan, Capps, and Nayga 2009). This adjustment was accomplished by adding a one-period lag term for both the right-hand side and left-hand side of equation [3.5]. It is necessary to estimate a common AR(1) coefficient among each of the equations in order to ensure that the adding-up restriction holds. The coefficient is estimated to be -.568 with a p-value of 0.000, which is statistically significant at the 1% level. The Barten decomposition parameter estimates (and p-values), δ_1 and δ_2, are estimated to be -.205 (.206) and -0.016 (0.941). The χ^2 statistics of the tests for the four hypothesized functional forms are given in TABLE 3.3. According to the four tests, only the Rotterdam model fails to reject its corresponding null hypothesis, suggesting that the data appear to fit the Rotterdam specification.

TABLE 3.3. Hypothesis Tests for Four Models

Null Hypothesis			*Chi-sq(2)*	*P-value*
(Rotterdam)	$\delta_1 = 0$	$\delta_2 = 0$	3.82	0.148
(LA-AIDS)	$\delta_1 = 1$	$\delta_2 = 1$	35.04	0.000
(CBS)	$\delta_1 = 1$	$\delta_2 = 0$	142.27	0.000
(NBR)	$\delta_1 = 0$	$\delta_2 = 1$	59.4	0.000

Price and expenditure elasticities derived from the Barten estimation procedure are provided in TABLE 3.4. It is important to note that the results reported in this table represent the within-group elasticities, which do not incorporate the indirect effect of price change on total group expenditure. The expenditure elasticities vary widely across the Texas games. Lotto Texas and the Pick3/Daily 4 composite both exhibit sub-unitary

TABLE 3.4. Barten Synthetic Demand System Price and Expenditure Elasticity Estimates

Uncompensated Price Elasticities

n = 385		Price				
		Lotto Texas	Mega Mil	Two Step	Cash 5	Composite
Quantity	Lotto Texas	**-1.096**	0.190	0.285	0.049	-0.171
		0.000	0.000	0.000	0.000	0.000
	Mega Mil	0.009	**-1.708**	0.505	0.085	-0.380
		0.754	**0.000**	0.000	0.000	0.000
	Two Step	0.304	1.485	**-6.294**	0.538	-0.979
		0.000	0.000	**0.000**	0.000	0.000
	Cash 5	0.045	0.320	0.512	**-3.015**	1.034
		0.016	0.000	0.000	**0.000**	0.065
	Composite	0.006	0.045	0.104	0.346	**-0.549**
		0.489	0.061	0.000	0.015	**0.003**

Compensated Price Elasticities

		Price				
		Lotto Texas	Mega Mil	Two Step	Cash 5	Composite
Quantity	Lotto Texas	**-0.906**	0.371	0.324	0.118	0.093
		0.000	0.000	0.000	0.000	0.001
	Mega Mil	0.389	**-1.345**	0.584	0.222	0.151
		0.000	**0.000**	0.000	0.000	0.003
	Two Step	1.568	2.689	**-6.033**	0.992	0.784
		0.000	0.000	**0.000**	0.000	0.000
	Cash 5	0.327	0.589	0.571	**-2.914**	1.427
		0.000	0.000	0.000	**0.000**	0.010
	Composite	0.067	0.103	0.116	0.368	**-0.464**
		0.001	0.003	0.000	0.010	**0.009**

Expenditure Elasticities

Lotto Texas	Mega Mil	Two Step	Cash 5	Composite
0.743	1.489	4.945	1.102	0.240
0.000	0.000	0.000	0.000	0.000

P-values are reported under each coefficient estimate

elasticity expenditure estimates, suggesting relatively low sensitivity to changes in expenditure levels. Mega Millions, Texas Two Step, and Cash 5 each report magnitudes above one, indicating a higher degree of sensitivity. These results suggest that as Texas players' budgets for lottery gambling change, they tend to remain relatively less loyal to Lotto Texas and Pick3/Daily4, on average.

The models presented in earlier studies that estimate price elasticities all report uncompensated measures, which incorporate income and substitution effects simultaneously. The estimates of the uncompensated Barten price elasticities, evaluated at the sample means for each game are also reported in TABLE 3.4. Own-price elasticities are marked in bold along the diagonal of the matrix. Each game exhibits elastic demand with the exception of the composite, which yields an inelastic own-price estimate. These estimates are all statistically significant at the 1% level. Texas Two Step and Cash 5 exhibit the greatest sensitivity to changes in their own respective effective prices, with the large magnitude for Texas Two Step's own-price elasticity likely due, in large part, to its relatively low budget share. This argument could also be made to explain the large value of Texas Two Step's expenditure elasticity as well.

The uncompensated cross-price elasticity estimates give insight into the degree of substitutability between games in the face of income effects. These values are reported in the off-diagonal cells in the table. Most of the cross-price terms are positive, suggesting that most of the games are gross substitutes for one another. The only negative cross-price values appear in the composite column. The table also reports the compensated elasticity matrix, which measures the pure substitution effects net of any income effects. These estimates exhibit a much larger degree of substitution among all five games. In fact, all games are net substitutes of each other at the mean and each estimate is statistically significant at the 1% level.

TABLE 3.5 reports the adjusted uncompensated price elasticities, which incorporate the indirect effect of price changes on total group expenditure under two-stage budgeting. The most important things to note are that each of the own-price elasticities is slightly higher in magnitude and that signs of each of the cross-price elasticities in the first column have reversed.

TABLE 3.5. Barten Uncompensated Price Elasticities Adjusted for Two-Stage Budgeting

n = 385		Price				
		Lotto Texas	Mega Mil	Two Step	Cash 5	Composite
Quantity	Lotto Texas	**-1.229**	0.063	0.257	0.001	-0.358
		0.000	0.001	0.000	0.926	0.000
	Mega Mil	-0.259	**-1.963**	0.450	-0.011	-0.754
		0.000	**0.000**	0.000	0.701	0.000
	Two Step	-0.587	0.637	**-6.478**	0.218	-2.221
		0.000	0.000	**0.000**	0.060	0.000
	Cash 5	-0.153	0.131	0.471	**-3.086**	0.757
		0.000	0.000	0.000	**0.000**	0.181
	Composite	-0.037	0.003	0.095	0.330	**-0.609**
		0.000	0.838	0.000	0.021	**0.002**

P-values are reported under each coefficient estimate

As demonstrated in Section 3.2, the total profit is maximized if the condition expressed in [3.22] holds. Using the values estimated in the Barten demand yields a value of -0.829. Testing the hypothesis that this value is equal to -1 results in a Chi-squared statistic (df = 1) of 258.01 with is statistically significant at the 1% level. This results leads to the rejection of the null hypothesis, providing strong evidence that the Texas Lottery's portfolio is not maximizing total average profit. This is most likely due in part to the substitutionary relationship between the games, whereby a substantial degree of sales cannibalization is occurring.

3.5 CONCLUSIONS

The aim of this study was to develop a model for lottery demand that controls for the direct cross-price effects of competing lottery games within a state's portfolio and examine whether the portfolio's pricing scheme leads to optimal profits. It offers a number of methodological contributions to the literature. First, it continues the work of developed in Section 2 by using an alternative effective price approach to model demand for several games from the perspective of consumers, thereby defining the effective price to be $p = P/EV$ allows for unrestricted use of observations with super-unitary expected values. Second, demand elasticities are estimated using a formal demand system, which is not only consistent with the theory of utility maximization, but allows estimation expenditure elasticities, both compensated and uncompensated own- and cross-price elasticities as well as provide identification for games that exhibit no price variation. Third, elasticity estimates are used to evaluate whether the portfolio's profits are maximized over all games. The major results of estimating the Barten demand system provide strong evidence that games within Texas' portfolio are all net substitutes of one another and mostly gross substitutes as well. Evaluating the profit maximization problem of the Texas Lottery reveals evidence that the games are not optimally priced, leading to sub-optimal profits over the sample period. This results is particularly interesting when considering the findings of previous studies. The models developed by Forrest, Gulley, and Simmons (2004), and Perez and Forrest (2010) report small and statistically insignificant cross-price estimates, suggesting that competing games tend to be largely independent of one another. Both Grote and Matheson (2006), and Purfield and Waldron (1999) report findings that suggest a slight complementary relationship between closely competing games.

The findings in this study also have important policy implications. First and foremost is that lottery games compete with each other and players are sensitive to relative price differences within a lottery market. This is result is critical for any lottery jurisdiction considering either altering the rules of existing games, adding new games to their portfolio, or joining a multi-state consortium. With such a large amount of money being generated through the sales lottery tickets, pricing policies based upon imprecise estimates of demand can likely lead to sizable losses in potential revenue. Cross-price effects matter, therefore analysis of profits based upon elasticity estimates obtained from modeling games in isolation is likely to be heavily biased. The result that profits are not maximized suggests that lottery portfolio operators may very well stand to benefit from re-evaluating their product mix and pricing.

Unfortunately, the elasticities estimated in this study only provide a piece of the complete dynamic system created by these repeatedly played games. Much of the how these games interact under different circumstances is not largely understood. Determining the best way to improve a portfolio’s pricing scheme is not an easy task. With limitations in available data, potential policy changes would likely need to be simulated in order to better understand the how profits streams can be increased. The development of a formal approach to address this issue is discussed in the following section.

4. IMPROVING PROFIT STREAMS OF STATE LOTTERIES THROUGH DYNAMIC EFFECTIVE-PRICE ANALYSIS

4.1 INTRODUCTION

In this essay I develop a new method to analyze the profitability of different pricing schemes that explicitly accounts for the intertemporal nature of lottery games that include the rollover of unclaimed jackpots. Previous work treats the profit maximization problem of the lottery controller in a static framework so that changes in price only influence profitability through their effect on period-by-period sales and expected payouts. This situation neglects, however, that changes in the probability that a jackpot is won influences the probability of reaching new drawings with higher jackpot amounts. For example, suppose that demand for a lotto game is inelastic at current prices, expected profitability of a given drawing decreases with jackpot size, and rollovers cause the subsequent jackpot to increase when there is no winning draw. An increase in the price of the game would increase first-period profit, but would also decrease total sales and thus increase the probability of reaching a drawing with a higher jackpot in the second period. Since expected profitability is decreasing in jackpot size, it is possible for the expected gain in first-period profit from increasing price to be partially offset by the decrease in expected second-period profit because the probability of reaching a drawing with lower expected profit has increased.

Thus, while there is little cost in viewing lotteries as repeated static games from the perspective of players, the problem of profit maximization is clearly connected intertemporally because of rollovers. Intuitively, the more the jackpot is rolled over, the higher the ticket sales, but the more the resulting payouts will be when the jackpot is eventually claimed. Because this issue cannot be directly addressed in a static framework, I utilize a Monte Carlo integration procedure to obtain a measure of expected profit through the simulation of lottery play over a period of four years. Such a procedure also provides a way to examine the effects of hypothetical changes in a game's pricing on total profits earned by the lottery controller.

The quality of this analysis depends entirely on the quality of information about the demand relationship for lotto games. Thus, in estimating the relationship between the price of lotto games and sales, which is used in the Monte Carlo simulation procedure, I incorporate the three innovations discussed in Section 2 of redefining the effective price to allow the inclusion of observations with super-unitary expected values, solving the problem of endogenous price by calculating the expected values using expected sales data, and estimating the demand curve using non-parametric local linear regression to avoid imposing the curvature restriction of iso-elasticity.

4.2 METHODOLOGY

The previous two sections describe the methodology for deriving the lottery controllers static maximization problem. While the consumer sees the lottery a repeated one-shot game, the lottery controller wishes to maximize a stream of profits over an extended period of time, where the expected profits in each period are related intertemporally. According to the static calculus, nothing matters beyond the period at hand, but in reality if a winner is not manifest then the lottery's payout obligation is not eliminated, but merely postponed one period. Therefore there is a cost of moving to the next period, in terms of a slightly larger payout, that must be weighed against the alternative state of having a winner chosen in the present period. While the realization of a winner in any given period is not a choice variable (but randomly determined), the lottery controller can influence how far the game tends to play out indirectly through its choice of the odds, takeout rate, and nominal price. In order to model the intertemporal costs and benefits of rolling jackpots, profit analysis should be conducted in a dynamic framework.

Further motivation to support a dynamic model is given by the added possibility of analyzing hypothetical policy changes. In addition to answering the question of "in which direction?" should prices change to increase profit, a dynamic model can provide insight into answering the question of "by how much?". A lottery controller has control over five different methods to alter the effective price of a lotto bet, which include changing the nominal ticket price, the odds of winning each prize tier, the specific amounts allocated to fund each prize tier, the number of prize tiers offered, and how to manage period-by-period rollovers. Each potential policy change will affect the outcome of a series of drawings in different ways. For example, effective April 26, 2006 the official rules for Lotto Texas were changed in three different directions in an attempt to increase ticket sales. First, the game moved from a double draw matrix of 5-of-44 and 1-of-44 to a single matrix of 6-of-54, effectively increasing the odds of winning the jackpot from 1:47,784,352 to 1:25,827,165. Second, it reduced the number of lower-tier prizes from 7 to 3. Third, the way the method for funding the prizes was restructured. Before the rule change, a percentage of Lotto Texas ticket sales were allocated to a general prize pool. Then percentages of this prize pool were then sub-allocated to each prize level, respectively. Currently (post rule change) the funds allocated to each prize level are directly a percentage of total ticket sales and allows the Texas Lottery Commission to use proceeds from other games to fund Lotto Texas prizes.

To illustrate the effects of this rule change on the Lotto Texas demand curve, effective prices and quantities are plotted for both time periods in FIGURE 4.1. The black squares provide the effective price/quantity relationship before the new rules were implemented and the gray diamonds denote the demand relationship under the new rules. There is a clear downward shift in the demand curve, demonstrating that altering the rules of the game can have a substantial impact on the demand relationship.

Since the extreme rarity of observable policy changes to a lottery game renders direct econometric analysis impossible, I propose the use of Monte Carlo integration methods to aid in the development of a dynamic model that simulates lottery play under a pre-determined set of rules over the course of several years. Then price parameters can then be adjusted to simulate hypothetical changes in the game's rules to model how individuals' ticket purchasing behavior would respond. Total simulated profits can be computed under the various policy changes and compared.

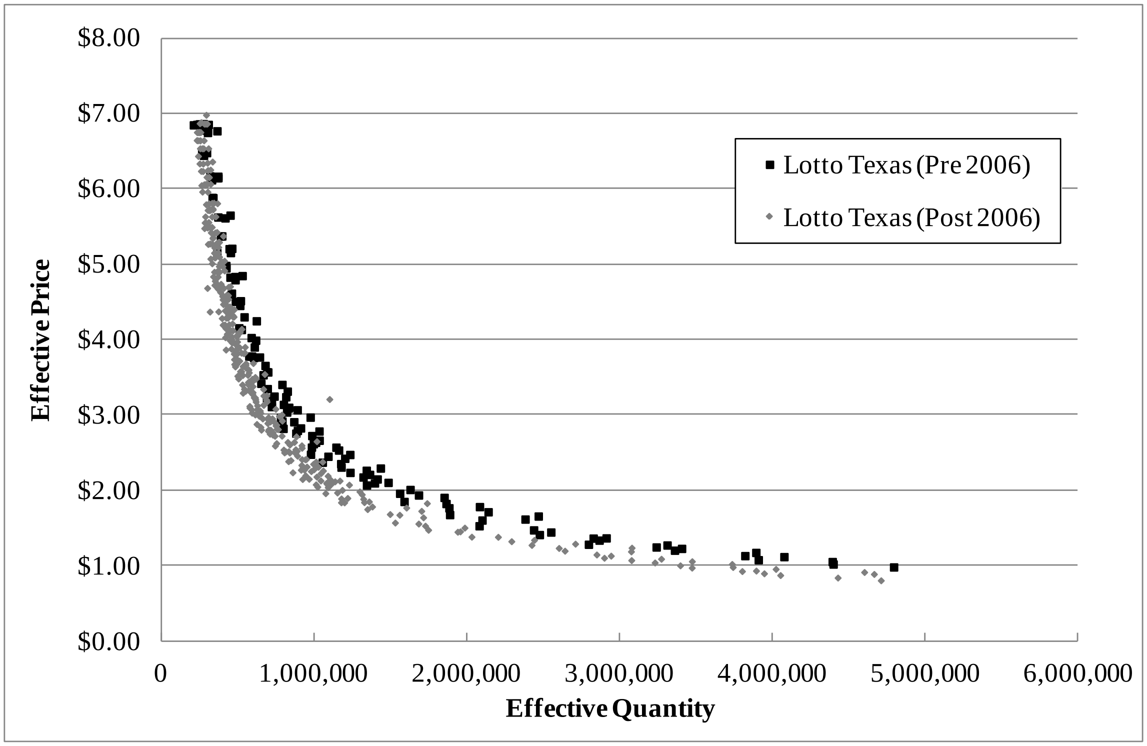

FIGURE 4.1. Lotto Texas Rule-Change-Induced Demand Curve Shift

Although this study is primarily concerned with the game Lotto Texas from an empirical standpoint, the methods discussed can be easily adapted to suit the specific characteristics of any lotto-style game. At its heart, the Monte Carlo procedure simulates 416 independent lottery drawings, which amount to 4 years of play at a rate of two drawings per week. This 4-year trial is then repeated 500 times with the total simulated profits from each trial averaged together to obtain a mean representative value. This method directly takes the static modeling techniques discussed above and applies them in a dynamic framework to capture the intertemporal cost of allowing the jackpot to be rolled over into future periods. Simulating lottery play not only provides a direct estimate of total profits, but also allows the analysis of hypothetical policy changes to examine whether profit streams can be increased.

First the initial values are chosen. For Lotto Texas, a roll cycle always begins with an advertised jackpot of $4,000,000, which provides a natural starting point to run the procedure. Since the simulation will run for exactly four years of lottery play, each trial is timed to begin in the month of January.

Second, the *ex ante* sales predictions are modeled according to a regression of Lotto Texas' total sales on the jackpot, month dummies, and jackpot/month interaction terms. The estimates were adjusted for first-order serial correlation using a Prais-Winsten regression with a Cochran-Orcutt transformation made available in the STATA statistical software package. The parameter estimates are provided in TABLE 4.1. The actual ticket sales prediction formula used by the TLC not publicly available so this formula has to be approximated. The month dummies are included to simulate seasonal variation in ticket sales. The predicted values of this regression enter into the calculation of the *ex ante* expected value along with the prize amounts and the respective probabilities of winning these prizes according to expression [2.11] in Section 2. The expected value is then converted to the effective price according to expression [2.3] in Section 2.

TABLE 4.1. Sales Prediction Regression Parameter Estimates

Dependent Variable: Ticket Sales			
N = 385	R-sq = .8764	D-W = 2.606	
Independent Variables:			
jackpot	0.0263 0.000	const.	1,616,923 0.000
jan	26,961.68 0.736	jan_int	-0.0013 0.788
feb	-52,602.11 0.549	feb_int	0.0057 0.157
mar	-20,198.07 0.745	mar_int	0.0095 0.004
apr	27,851.74 0.621	apr_int	0.0047 0.169
may	231,240.10 0.005	may_int	-0.0168 0.026
jun	-83,702.61 0.273	jun_int	0.0070 0.186
jul	162,131.80 0.020	jul_int	-0.0114 0.011
aug	-44,880.80 0.546	aug_int	0.0025 0.510
sep	-29,766.23 0.603	sep_int	0.0070 0.035
oct	-34,945.95 0.534	oct_int	0.0035 0.279
nov	65,523.00 0.346	nov_int	-0.0043 0.376

P-values are reported under each coefficient estimate

Third, *ex post* actual sales are obtained from the fitted values of a non-parametric LOWESS regression of log-effective sales on log-effective price. More specifically, the LOWESS estimation provides a grid of price/quantity pairs, so given a value for the *ex ante* effective price in the simulation, the procedure finds the closest matching price value in the LOWESS grid and assigns the corresponding fitted quantity, from which the *ex post* total ticket sales can be obtained. It is important to note that the LOWESS fitted values can only be obtained for prices within the observable range. Since the random assignment of values from the uniform distribution could potentially lead to longer strings of simulated rollovers than is actually observed in the underlying data sample, some extrapolation is necessary. For example, in the Lotto Texas data sample the longest string of rollovers was 47 draw periods, resulting in the lowest observed effective price of $0.54. Running the simulation under 500 trials will likely result in strings of rollovers greater than 47, resulting in prices with no matching LOWESS fitted values for quantity.

In order to solve this problem, I chose to extrapolate linearly beyond the two ends of the Lotto Texas demand curve, as illustrated in FIGURE 4.2. Effective quantity is linearly regressed on effective price using two separate subsamples at the tails of the demand curve. The slope coefficient is used to extend the quantity prediction for any simulated points beyond the observable range. Because I re-evaluate the integration under various hypothetical nominal prices, a greater number of simulated points will need to be extrapolated. As a result, I feel comfortable only allowing a maximum of 10% of the simulated points to extend beyond the observable range.

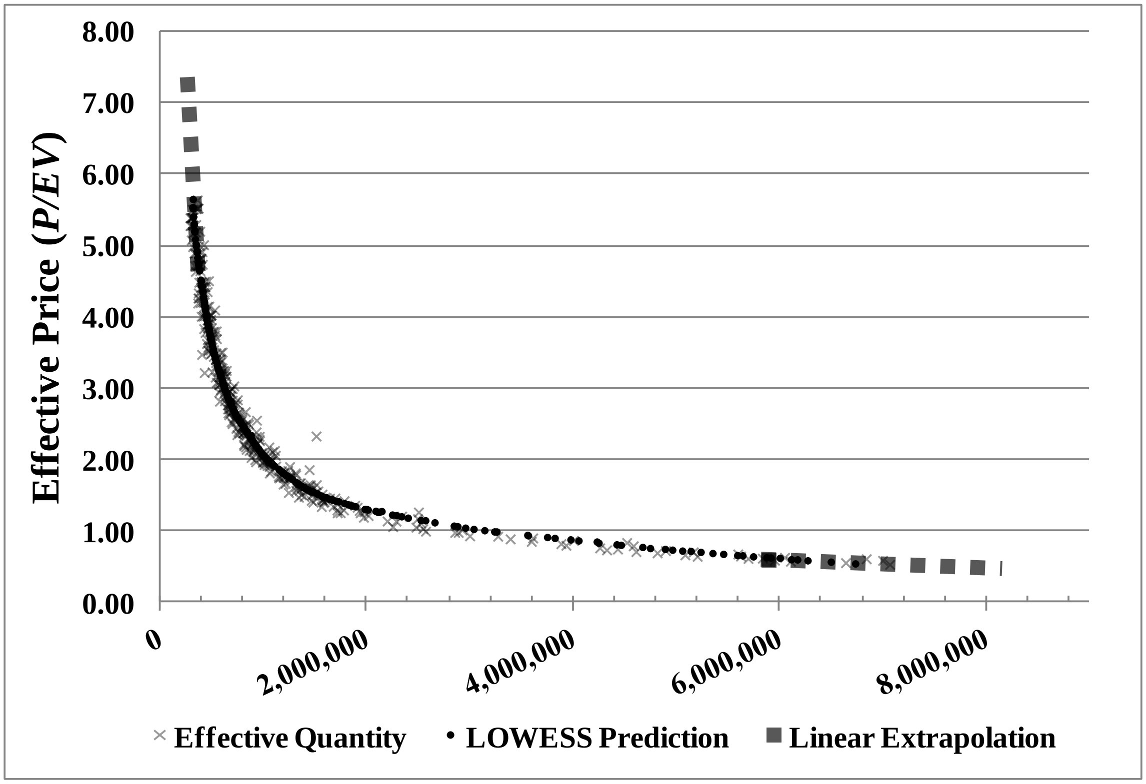

FIGURE 4.2. Linear Extrapolation of the Ends of the Lotto Texas Demand Curve

Fourth, once total sales have been determined it becomes necessary to establish the transition into the next period. This situation will result in two potential states: (1) the jackpot is won and reset to \$4 million, or (2) the jackpot is not won and consequently rolled over. In reality, the realization of a winning ticket for any given drawing is determined by the matching of a player's chosen numbers to the official randomly-drawn numbered balls. Operating under the assumption that the numbers players choose are uniformly distributed across the set of all players, Farrell, Morgenroth, and Walker (1999) point out that the probability α of a rollover for a m-of-n game, given total ticket sales N can be modeled by

$$\alpha = \left[1 - \frac{m!(n-m)!}{n!}\right]^N . \qquad [4.1]$$

Therefore in order to simulate the potential for a rollover in a given drawing, α is computed according to [4.1] and then compared to a randomly drawn value obtained from a uniform distribution over the interval [0, 1], call λ. If $\alpha \geq \lambda$, the jackpot is won and reset to the base, while payouts for lower-tier prizes are assigned according to their the size of their prize pools. If $\alpha < \lambda$, then a rollover is triggered and the new jackpot must be determined. According to the official rules for Lotto Texas, the jackpot prize must be the greater of: (1) 40.47% of the proceeds from Lotto Texas ticket sales and any earnings on an investment of all or part of the proceeds from tickets sales, paid in annual installments; or (2) the amount advertised, paid in 25 annual installments.

In practice, the advertised jackpot for Lotto Texas exceeds 40.47% of total tickets sales due in part to stochastic variation, but also by the TLC's option to fund the Lotto Texas jackpot using outsides sources, most likely exercised to attract higher-than-normal ticket sales. Clause 1 is a less-generous approach and provides a lower (upper) bound on the jackpots paid out (profits brought in). Similarly, Clause 2 is more generous to players and provides an upper (lower) bound on the jackpots paid out (profits brought in). The simulation is run under both rollover policies to provide a range of feasible values. It is important to note that because earnings on investment are impossible to determine without intimate knowledge of the TLC's investment strategies, I assume that the amount added to the previous period's jackpot can be determined by calculating 40.47% of sales and rounding to the nearest $1 million. The TLC has always rounded to the nearest million in their own practice so I argue that my approximation is a reasonable approach. The payouts for the lower-tier prizes are awarded regardless of whether the jackpot is rolled. The second- and third-tier prizes are pari-mutuel, with respective prize pools of 2.23% and 3.28% of sales. The fourth-tier prize is a guaranteed $3, so the simulated prize payout at this tier is simply the prize amount multiplied by the probability of winning multiplied by the total number of sales. Finally, total payouts are subtracted from the total ticket revenue for each drawing. The resulting profit then is summed over each of the 500 trials and averaged together to obtain an estimate of four years worth of total profits.

Unfortunately the Lotto Texas data are limited in their ability to provide information that will aid the analysis of hypothetical changes in the lotteries operational policy. According to the effective price model, the observed variation in the effective price caused by rollovers only indentifies movement along the demand curve and provides nothing to help identify potential shifts in demand. Even using the results of the 2006 rule change in the form of a natural experiment would not be of much use since the rule change itself was multi-faceted and it is impossible to separately identify the individual effects of each piece. However, this situation does not prevent further exploration of potential improvements in profitability as a result of policy changes that affect positioning *along* the demand curve.

The demand curve of a lotto game is quite asymmetric. Most observations occur near the base jackpot since it is to this point that the game always resets following a winning draw. In this region, the number of tickets sales is low, but the expected profit per ticket is high. On the other side, we observe a lower effective price with much fewer observations, but the number of sales is much higher. Since the two sides of the demand curve are considerably different, it is possible to force the game to play out more frequently to one side or the other, potentially resulting in higher profit streams. Essentially this can be accomplished by altering the pricing of the game to impose movement along the curve *en masse*. The most straightforward approach is to simulate changes in the nominal price of a ticket. Altering the nominal price of a ticket imposes two separate effects on the profitability of the game. First is a direct effect on the effective price of a bet according (see expression [2.3]), which causes the game to play out along a different section of the demand curve. Second is an indirect effect on the distribution of jackpots the lottery controller will ultimately pay out. Changing the effective price of a ticket will impact total sales, which in-turn impacts the probability that a jackpot is won, which in-turn influences the probability of reaching new drawings with higher jackpot amounts. Static profit analysis does not provide a pathway to explore this indirect effect, further supporting the need to employ dynamic methods.

4.3 DATA

The data for this study were collected from public records provided by the Texas Lottery Commission (TLC) and include daily sales, advertised jackpots, and odds at each prize tier for the games Lotto Texas spanning April 2006 to year’s end 2009. This information was also collected for the games Texas Two Step and Mega Millions, which are also offered by the TLC with effective prices that enter the model as a control for competing games. Since the data used in this study are identical to those used described in Section 2, the reader is invited to refer to TABLE 2.1 as well as the corresponding description.

4.4 RESULTS

The results for the Monte Carlo procedure are provided in TABLE 4.2, total simulated profits were obtained under nominal prices ranging from $0.70 to $1.40 in increments of $0.10 for two separate rollover schemes. The bounds set on the nominal price change correspond to the extrapolation restriction requiring less than 10% of simulated points to fall outside of the observable price range. Changing the nominal price makes a suitable policy option to analyze because it has a direct effect on the effective price of a ticket, inducing movement along the demand curve. Lotto Texas currently is nominally priced at $1, which provides a benchmark for comparison. According to TABLE 4.2**,** a price of $1.00, the total number of tickets sold range from 1.04 billion to 1.06 billion, depending on the level of generosity of the rollover policy. Also at this price, profits (i.e. revenue net of payouts) amount between $566 million to $665 million over the course of four years, or a takeout rate range of 53.2% to 64.0%. With the number of actual Lotto Texas tickets sold by the TLC in 2010 amounting to 256 million, the simulated average of approximately 263 million tickets per year appears to be quite reasonable in magnitude, lending support to the predictive power of the simulation model.

Simulated changes in the nominal price reveal a monotonic increase (decrease) in total profit as the nominal price rises (falls). This result suggests that according to the Monte Carlo procedure, elasticity of total profit with respect to the sales is inelastic over the simulated nominal price range. Thus a hypothetical increase in the nominal ticket price of \$0.40 is estimated to result in a total increase in profit ranging from \$142 million to \$191 million. In percentage terms, 40% increase in the nominal price is estimated to resulting in a percentage profit increase of 25% to 29%, again depending on level of rollover generosity. To put this value into perspective, according to the Texas Comptroller of Public Accounts, the total budget for education in Texas was \$57.9 billion for the 2008-2009 school year. Proceeds from the Texas Lottery contributed to 1.7% of the budget, or \$984 million. This result indicates that a price increase of \$0.40 from this one lottery game alone would provide a 3.6% - 4.8% increase in lottery's total contributions to Texas' education fund.

TABLE 4.2. Results of Simulation over Various Nominal Prices

Duration: 416 periods (4 years), Trials: 500

METHOD 1 - Rollovers computed as 40.47% of proceeds from Lotto Texas Ticket Sales (More conservative rollovers)

Nominal Price	*Tickets Sold*	*Revenue*	*Profit*	***Takeout Rate***	**Δ in Profit** *($1.00 base)*	**%Δ in *Sales*** *($1.00 base)*	**%Δ in Profit** *($1.00 base)*
$0.70	1,191,400,000	$834,000,000	$481,900,000	57.8%	-$182,900,000	14.7%	-27.5%
$0.80	1,130,200,000	$904,130,000	$544,900,000	60.3%	-$119,900,000	8.8%	-18.0%
$0.90	1,080,400,000	$972,350,000	$605,800,000	62.3%	-$59,000,000	4.0%	-8.9%
$1.00	**1,039,100,000**	**$1,039,100,000**	**$664,800,000**	**64.0%**	**---**	**---**	**---**
$1.10	1,003,500,000	$1,103,800,000	$715,040,000	64.8%	$50,240,000	-3.4%	7.6%
$1.20	972,830,000	$1,167,400,000	$759,380,000	65.0%	$94,580,000	-6.4%	14.2%
$1.30	946,860,000	$1,230,900,000	$807,060,000	65.6%	$142,260,000	-8.9%	21.4%
$1.40	925,430,000	$1,295,600,000	$855,570,000	66.0%	$190,770,000	-10.9%	28.7%

METHOD 2 - Rollovers computed based upon estimates of Lotto Texas' common practice (More generous rollovers)

Nominal Price	*Tickets Sold*	*Revenue*	*Profit*	***Takeout Rate***	**Δ in Profit** *($1.00 base)*	**%Δ in *Sales*** *($1.00 base)*	**%Δ in Profit** *($1.00 base)*
$0.70	1,204,000,000	$842,810,000	$437,410,000	51.9%	$128,450,000	13.2%	-22.7%
$0.80	1,147,200,000	$917,780,000	$471,610,000	51.4%	$94,250,000	7.9%	-16.7%
$0.90	1,101,000,000	$990,930,000	$517,900,000	52.3%	$47,960,000	3.5%	-8.5%
$1.00	**1,063,400,000**	**$1,063,400,000**	**$565,860,000**	**53.2%**	**---**	**---**	**---**
$1.10	1,033,100,000	$1,136,400,000	$616,580,000	54.3%	$50,720,000	-2.8%	9.0%
$1.20	1,020,100,000	$1,224,100,000	$652,460,000	53.3%	$86,600,000	-4.1%	15.3%
$1.30	1,009,000,000	$1,311,600,000	$687,400,000	52.4%	$121,540,000	-5.1%	21.5%
$1.40	1,018,900,000	$1,426,400,000	$707,880,000	49.6%	$142,020,000	-4.2%	25.1%

The Monte Carlo simulation also allows the ability to look at the analysis in terms of profit elasticities. According to the model, a 1% increase in the nominal price of a ticket would lead to an estimated profit increase of either 0.76% (under conservative rollovers) or 0.96% (under generous rollovers). The computation of these values is provided in TABLE 4.3. There is quite a large contrast when comparing these elasticities to those estimated under the static model outlined in Section 2. The static model elasticity estimates are also reported in table. While the profit elasticity under the dynamic model is straight forward to calculate, the profit elasticities reported for the static models of Section 2 need to be derived from their respective estimated price elasticities. TABLE 4.4 reports the effective-price and profit elasticities for each of the static models discussed in Section 2 as well as the profit elasticities obtained from the dynamic model.

The price elasticity obtained under the effective price defined by expression [2.1] (see Section 2) is given by

$$\eta_{P-EV} = \frac{\%\Delta Q}{\%\Delta\left(P-EV\right)}. \qquad [4.2]$$

Under this effective price definition, one cannot include the observations with super-unitary observations into the log-log EPM. The percentage change in profit as a function of ticket sales (*Q)* and the average profit per ticket (*A* = *P* – *EV)* is given by

$$\%\Delta\pi = \%\Delta Q + \%\Delta(P-EV). \qquad [4.3]$$

Substitution of [4.2] into [4.3] yields the profit elasticity equation, under an effective price of *P* – *EV*, or

TABLE 4.3. Profit Elasticities under the Dynamic Model

METHOD 1 - (Conservative rollovers)

Nominal Price	*Tickets Sold*	*Revenue*	*Profit*	**Δ in Profit**	**%Δ in Profit**
$1.00	1,039,100,000	$1,039,100,000	$664,800,000	---	---
$1.01	1,034,600,000	$1,045,000,000	$669,820,000	$5,020,000	**0.76%**

METHOD 2 - (Generous rollovers)

Nominal Price	*Tickets Sold*	*Revenue*	*Profit*	**Δ in Profit**	**%Δ in Profit**
$1.00	1,063,400,000	$1,063,400,000	$565,860,000	---	---
$1.01	1,058,800,000	$1,069,400,000	$571,290,000	$5,430,000	**0.96%**

TABLE 4.4. Effective-Price and Profit Elasticity Comparison:
Static vs. Dynamic Models

Model	**Price Elasticity (η)**	**%Δ Profit**
Log-log ($p = P - EV$)		
Omitted	-0.166	**1.46%**
Censored	-0.145	**1.72%**
Unrestricted	---	---
Log-log ($p = P/EV$)		
Omitted	-1.201	**0.54%**
Censored	-1.403	**0.61%**
Unrestricted	-1.302	**0.97%**
Semi-log ($p = P - EV$)		
Omitted	-0.239	**1.33%**
Censored	-0.331	**1.35%**
Unrestricted	-0.249	**1.71%**
LOWESS		
$p = P\text{-}EV$	-0.205	**1.56%**
Dynamic		
Conservative rollovers	---	**0.76%**
Generous rollovers	---	**0.96%**

$$\%\Delta\pi = \%\Delta(P - EV) * (1 + \eta). \qquad [4.4]$$

In order to determine the profit elasticity, it is necessary to evaluate expected value at the sample mean. The sales-weighted mean expected values ($\overline{EV}$) under the three samples: (1) omitted observations, (2) censored observations, and (3) unrestricted observations are 0.427, 0.503, and 0.560, respectively. According to the log-log specification with a defined effective price of *\$1-EV*, the estimated price elasticities are η_{omit} = -0.166 and η_{censor} = -0.145, which imply that a 1% increase in the price of a ticket will result in a profit increase of 1.41% and 1.57%, respectively. According to the semi-log specification with the same effective price definition, the estimated price elasticities are η_{omit} = -0.239, η_{censor} = -0.331, and $\eta_{unrestr}$ = -0.249, which imply that a 1% increase in the price of a ticket will result in a profit increase of 1.33%, 1.35%, and 1.71%, respectively. Finally under the LOWESS model, the sales-weighted mean arc-elasticity is η_{LOWESS} = -0.205, which implies a profit elasticity of 1.56.

Converting the price elasticities obtained from the log-log model under the effective price definition *p = P/EV*, requires a different procedure. The price elasticity is given by

$$\eta_{P/EV} = \frac{\%\Delta q}{\%\Delta\left(P/EV\right)}, \qquad [4.5]$$

where *q = Q*EV*. Algebraically rewriting [4.5] reveals the following expression:

$$\eta_{P/EV} = \frac{\%\Delta Q + \%\Delta EV}{\%\Delta P - \%\Delta EV}. \qquad [4.6]$$

A change the effective price of a ticket would lead to a change in the total expected sales for a given drawing, which would in-turn affect the magnitude of the expected value, which again would change the effective price, and so on. To address this issue, I simulate this iterative effect at a number of different points along the demand curve and find that the effect of the price change on *EV* under the first iteration is so small that it makes absolutely no impact whatsoever and can be ignored entirely. Thus the $\%\Delta EV$ terms in expression [4.6] reduce to zero, resulting in

$$\eta_{P/EV} = \frac{\%\Delta Q}{\%\Delta P}. \qquad [4.7]$$

Substituting expression [4.7] into [4.3] provides the profit elasticity equation under an effective price of *P/EV*, or

$$\%\Delta\pi = \eta(\%\Delta P) + \%\Delta(P - EV). \qquad [4.8]$$

Therefore, given the same sales-weighted mean expected values and price elasticities under the three sampling cases, or η_{omit} = -1.201, η_{censor} = -1.403, and η_{all} = -1.302, we obtain the respective profit elasticities of 0.54, 0.61 and 0.97.

Comparison of the dynamically estimated profit elasticities with the statically estimated profit elasticity groups reveals two interesting results. First, the static EPM under an effective price of *P–EV* tends to overestimate the change in profit. Second, the static EPM under an effective price of *P/EV* tends to underestimate the change in profit. This inconsistency, coupled with the inconsistency of the elasticity estimates over the three sample cases is likely best explained by the fact that the static model relies entirely upon mean-valued estimates, while the values estimated under the dynamic model take all observations into account.

Finally, in order to examine the intertemporal effects that rollover-induced changes in the effective price have on the distribution of jackpots, I compare the distributions of a subsample of the total number of simulated jackpot values under prices of \$0.70, \$1.00, and \$1.40. Again, the impact of changes in the effective price, which affect the realization of sales, which in-turn impacts the probability of reaching higher, more-costly jackpots cannot be modeled in a static framework. For each trial of 416 draw periods, 54 jackpots were randomly selected with replacement resulting in three subsamples of 27,000 observations corresponding to each of the nominal price policies.

A histogram of these subsampled (Poltis, Romano, and Wolf 1999) jackpot values for each of the three policies are provided in **FIGURE 4.3**. As predicted, higher effective prices lead to fewer sales, resulting in more frequent realizations of larger jackpots since the probability the jackpot gets rolled over increases. According to the three histograms, we observe a greater number of jackpots in the low-jackpot ranges (i.e. \$4 million to \$33 million) under the nominal price policy of \$0.70. Under a nominal price of \$0.70, all effective prices are thus comparatively lower than those realized under the other two nominal price policies. Similarly, under a nominal price \$1.40 (with comparatively higher overall effective prices) we observe a greater number of jackpots occurring in the high-jackpot ranges (i.e. \$73 million to \$133 million). These distributions were compared statistically, in pairs, using a Wilcoxon signed-rank test (Wilcoxon 1945). The null hypothesis is that the median difference between each pair of observations is zero. The p-values for each of the three tests (i.e. \$0.70 vs. \$1.00; \$0.70 vs. \$1.40; and \$1.00 vs. \$1.40) were all 0.000, which leads to the rejection of the null hypothesis. This result provides additional evidence to support the need for dynamic modeling framework for lottery profit analysis.

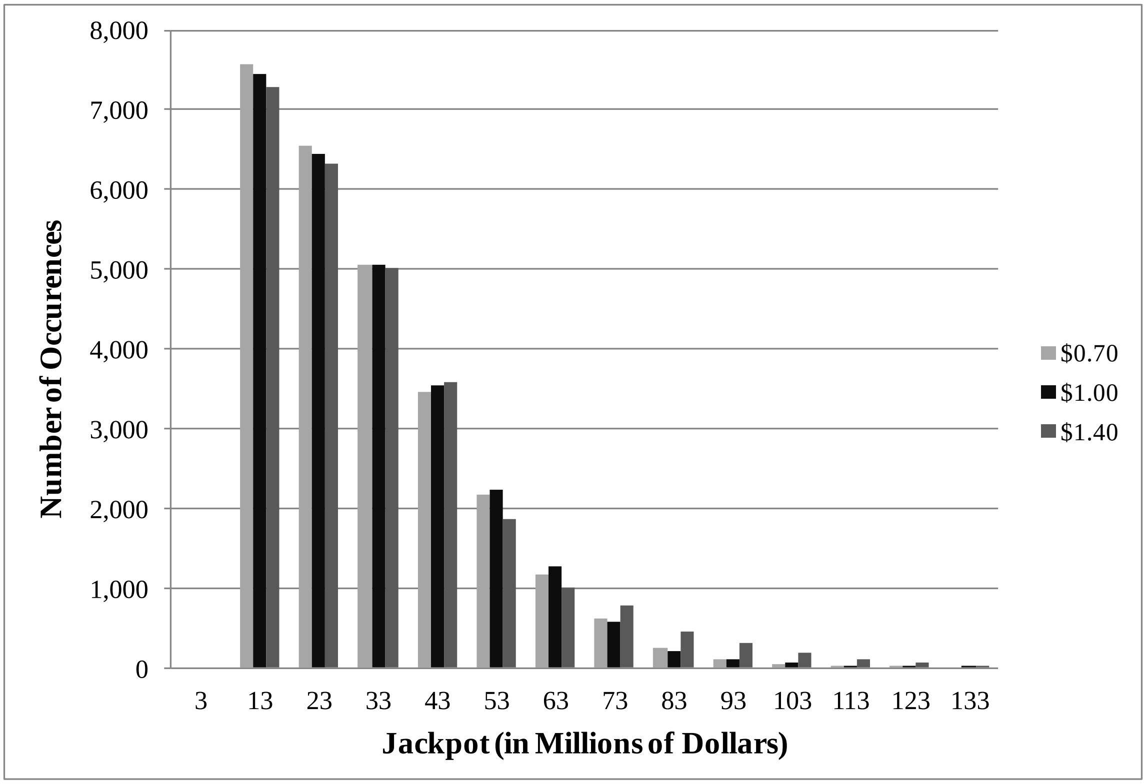

FIGURE 4.3. Distribution of Simulated Jackpot Frequency under Three Different Nominal Prices

4.5 CONCLUSIONS

This paper provides compelling evidence to support the need for a dynamic framework for the analysis of a lottery controller's profit stream. The traditional static approach provides a convenient, though limited interpretation of estimated lottery price elasticities. First the relationship between price and quantity cannot be meaningfully summarized using a single elasticity in the face of such a large degree of variation in the effective price. Furthermore, evaluating the static optimization conditions at every realized effective price still does not incorporate the intertemporal costs of a game moving to the next period via a rollover. To address this issue, the implications of the lottery's price on total profit are obtained through a Monte Carlo integration procedure. Results of the simulation under hypothetical changes in Lotto Texas' nominal price indicate that total profit can be raised by increasing the nominal price.

More specifically, a $0.40 increase in the nominal price of a Lotto Texas ticket would lead to an estimated increase in profit in the range of $142 million to $191 million over the course of four years. Since the simulated model does not take into consideration variable costs associated with operating a state-wide lottery game as well as potential substitution effects associated with a general price increase, these estimates provide a reasonable upper bound on the potential profit increase. With lotto games operating in similar fashion throughout the world, the methods and implications of this study can easily be extended to suit these other games as well.

This paper provides a significant contribution to the existing literature on lottery demand by providing a new perspective on how to analyze the pricing of lottery games. Topics for future research include the extension of the dynamic model across a portfolio of games operated simultaneously within a state. This situation also suggests a natural extension to the analysis discussed in Section 3, which provided evidence to suggest that competing games within a portfolio experience substantial substitution effects. Exploring how variation in the effective prices of the games impacts consumer purchasing behavior in a dynamic framework appears to be the next logical step.

5. SUMMARY AND CONCLUSIONS

In this dissertation, I have discussed in detail the work of three different, yet closely related studies on the demand and optimal pricing of state lotteries. This topic is particularly enlightening due to the recent trends showing increased reliance of state governments upon lottery financing as an alternative source for funding public programs. This body of work provides a notable contribution to the economic literature on lottery demand and profitability analysis, aiming to not only improve upon current methods for estimating the demand for lottery tickets, but also to develop new methods that can provide better insight into how best to adjust a game's structure in order to increase its profit stream.

In Section 2, I addressed and resolved three outstanding issues in the literature on lottery demand. First, I demonstrated how to incorporate observations with super-unitary expected values into the effect price model through the use of two alternative modeling strategies: (1) estimating a semi-log regression model and (2) redefining the effective price. By modeling demand from the perspective of consumer who purchases chances to win a dollar, I am able to include all available data into a log-log model without the need to censor or omit observations. Second, I introduce a new method for addressing the endogeneity of the effective, which takes advantage of publicly available sales prediction data, effectively eliminating the need to run a two-stage regression model. Third, I challenge the idea of using a mean-evaluated price elasticity measure to describe price sensitivity along the entire demand cure. Since the effective price exhibits such a high degree of variation over time, a single elasticity may obscure the natural curvature of the demand relationship. Ultimately, I conclude that the empirical results indicate that according to the Lotto Texas data, the bias induced by censoring extreme observations is greater than that of failing to control for endogeneity by a factor of six. Furthermore, using a log-log functional form for the demand relationship imposes excessive restrictions on its curvature, specifically iso-elasticity. By using a non-parametric local-linear regression approach, which does not impose a specific functional form, I show that estimates of demand elasticity actually vary along the demand curve. This finding suggests that simply looking at the mean provides too little information about the price sensitivity of demand among jackpots of different size.

In Section 3, I extend this analysis by developing a model to fit an entire portfolio of lottery games. Since states typically operates multiple games simultaneously, it is reasonable to suspect that a portfolios games could likely compete with one another for a player's dollar. Using Barten's synthetic demand system and incorporating modeling innovations introduced in Section 2, I estimate cross-price effects of portfolio of Texas lottery games and examine whether the games are individually priced scheme optimize profits over the whole portfolio. The results provide strong evidence that games within Texas' portfolio are largely substitutes of one another. Evaluating the profit maximization problem of the Texas Lottery reveals evidence that the games are not priced to maximize total portfolio profits.

In Section 4, I argue that profitability analysis of lottery games with rolling jackpots is better suited under a dynamic framework, which takes into account the intertemporal costs of being forced to postpone paying the jackpot in the event a rollover is triggered. To address this issue, the implications of the lottery's price on total profit are obtained through a Monte Carlo integration procedure. Results of the simulation under hypothetical changes in Lotto Texas' nominal price indicate that total profit can be raised by increasing the nominal price. More specifically, a $0.40 increase in the nominal price of a Lotto Texas ticket would lead to an estimated increase in profit in the range of $142 million to $191 million over the course of four years. Comparison of the profit elasticity under the dynamic model with those estimated using the static EPM reveals that the static EPM using an effective price of *P–EV* overestimates the profit elasticity, while the static EPM using an effective price of *P/EV* underestimates the profit elasticity. Furthermore examination of the jackpot distribution under various nominal prices provides evidence to support the need to model profitability in dynamic framework. Increasing the price of a ticket will lead to fewer sales, which increases the likelihood of reaching higher jackpots. This effect cannot be controlled for in the static framework.

The methods outlined in the previous sections do exhibit a few limitations that motivate the need for future work on this topic. First, the effective price model heavily upon the first moment of the payout distribution. It is possible that higher-order moments could play a significant part in the behavioral demand response to the relative differences in the period-by-period states of a lottery game. Incorporating the risk level of a bet beyond its expected value could help explain observable differences between games operated at different scales and with widely differing rules. Second, lottery operating states to engage in marketing and advertising to promote their games. Costs related to advertising and promotions were not explicitly model in this framework and thus could have some effect on the prediction made in the analysis. Third, including the sale of scratch-off (instant win) tickets into the analysis could have a significant impact on the estimation results. Sales of these tickets make up the largest share of revenue for the Texas Lottery. However, including these tickets into the model poses a significant challenge from an econometric perspective due to their short lifespan and lack of price variation over time. Another important topic for future research would entail the expansion of a Monte Carlo procedure to explore how profit streams are affected by policies other than a simple nominal price change, including changes in a game's odds, the distribution of lower-tier prizes, or even the potential of imposing a policy of nominally pricing in relation to the size of the jackpot. There is still much that we do not understand about the nature of inherently risky goods and further study on this topic may be able to provide important insight into other risk markets such as insurance or financial asset pricing as well.

REFERENCES

Barnett, William. 1979. “Theoretical Foundations for the Rotterdam Model.” *The Review of Economic Studies* 46(1): 109-130.

Barten, Anton. 1964 “Consumer Demand Functions under Conditions of Almost Additive Preferences.” *Econometrica* 32(1): 1–38.

Barten, Anton. 1993. “Consumer Allocation Models: Choice of Functional Form.” *Empirical Economics*, 18: 129–58.

Brown, Mark, Jonq-Ying Lee, and James Seale, Jr. 1994. “Demand Relationships among Juice Beverages: A Differential Demand System.” *Journal of Agricultural and Applied Economics* 26(2): 417–29.

Cleveland, William. 1979. “Robust Locally Weighted Regression and Smoothing Scatterplots.” *Journal of the American Statistical Association* 74(368): 829–836.

Cleveland, William, and Susan Devlin. 1988. “Locally-Weighted Regression: An Approach to Regression Analysis by Local Fitting.” *Journal of the American Statistical Association* 83(403): 596–610.

Clotfelter, Charles, and Philip Cook. 1989. *Selling Hope: State Lotteries in America.* Cambridge, MA: Harvard University Press.

Clotfelter, Charles, and Philip Cook. 1993. “The Peculiar Scale of Lotto.” *American Economic Review* 83(3): 634–43.

Deaton, Angus, and John Muellbauer. 1980. “An Almost Ideal Demand System.” *American Economic Review* 70(3): 312–26.

Farrell, Lisa, Roger Hartley, Gauthier Lanot, and Ian Walker. 2000. "The Demand for Lotto: The Role of Conscious Selection." *Journal of Business and Economic Statistics* 18(2): 228–241.

Farrell, Lisa, Edgar Morgenroth, and Ian Walker. 1999. "A Time Series Analysis of U.K. Lottery Sales: Long and Short Run Price Elasticities." *Oxford Bulletin of Economics and Statistics* 61(4): 513–26.

Farrell, Lisa, and Ian Walker. 1999. "The Welfare Effects of Lotto: Evidence from the UK." *Journal of Public Economics* 72: 99–120.

Forrest, David, O. David Gulley, and Robert Simmons. 2000. "Elasticity of Demand for UK National Lottery Tickets." *National Tax Journal* 53(4): 853–63.

Forrest, David, O. David Gulley, and Robert Simmons. 2004. "Substitution between Games in the UK National Lottery." *Applied Economics* 36: 645–51.

Forrest, David, O. David Gulley, and Robert Simmons. 2010. "The Relationship between Betting and Lottery Play." *Economic Inquiry* 48(1): 26–38.

Garrett, Thomas, and Thomas Marsh. 2002. "The Revenue Impacts of Cross-Border Lottery Shopping in the Presence of Spatial Autocorrelation." *Regional Science and Urban Economics* 32: 501–519.

Garrett, Thomas, and Russell Sobel. 2004. "State Lottery Revenue: The Importance of Game Characteristics." *Public Finance Review* 32(3): 313–30.

Grote, Kent, and Victor Matheson. 2006. "Dueling Jackpots: Are Competing Lotto Games Complements or Substitutes?" *Atlantic Economic Journal* 34: 84–100.

Gulley, O. David, and Frank Scott. 1993. "The Demand for Wagering on State-Operated Lotteries." *National Tax Journal* 45(1): 13–22.

Keller, W., and J. van Driel. 1985. "Differential Consumer Demand Systems." *European Economic Review* 27: 375–390.

La Fleur, Teresa, and Bruce La Fleur. 2010. *La Fleur's 2010 World Lottery Almanac*. Rockville, MD: TLF Publications, Inc.

Matsuda, Toshinobu. 2005. "Differential Demand Systems: A Further Look at Barten's Synthesis." *Southern Economic Journal* 71(3): 607–19.

Mountain, Dean. 1988. "The Rotterdam Model: An Approximation in Variable Space." *Econometrica* 56(2): 477-484.

Neves, P. 1987. "Analysis of Consumer Demand in Portugal, 1958-1981," *Memoire de Maitrise en Sciences Eonomiques*. University Catholique de Louvain.

Perez, Levi, and David Forrest. 2011. "Own- and Cross-Price Elasticities for Games within a State Lottery Portfolio." *Contemporary Economic Policy* 29(4): 536–549.

Politis, Dimitris, Joseph Romano, and Michael Wolf. 1999. *Subsampling.* New York: Springer-Verlag.

Pollock, Robert, and Terrance Wales. (1992). *Demand System Specification and Estimation*. Oxford: Oxford University Press.

Purfield, Catriona, and Patrick Waldron. 1999. "Gambling on Lotto Numbers: Testing for Substitutability or Complementarity Using Semi-Weekly Turnover Data." *Oxford Bulletin of Economics and Statistics* 61(4): 527–44.

Scoggins, John. 1995. "The Lotto and Expected Revenue." *National Tax Journal* 48(1): 61–70.

Stone, Richard. 1953. *The Measurement of Consumers' Expenditure and Behaviour in the United Kingdom, 1920–1938, Vol. 1*. Cambridge: Cambridge University Press.

Theil, Henri. 1965. "The Information Approach to Demand Analysis." *Econometrica* 33(1): 67–87.

Wilcoxon, Frank. 1945. "Individual Comparisons by Ranking Methods." *Biometrics Bulletin*, 1(6): 80-83.

Yuan, Yan, Oral Capps, Jr., and Rodolfo Nayga. 2009. "Assessing the Demand for a Functional Food Product: Is There Cannibalization in the Orange Juice Category?" *Agricultural and Resource Economics Review*, 38(2): 153-165.

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